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В качестве последнего примера рассмотрим 2-однородный граф, имеющий двумерную пространственную группу симметрии $pgg2$. Его код в базе RSCR [7] — krj. Он также имеет две симметрически независимые вершины v_1 и v_2, обе вершины степени 5. Его координационные последовательности имеются в OEIS [8] под номерами A219529 и A301697, соответственно. Фрагмент этого графа изображен на рис. 2. б).

Для графа krj справедлива следующая теорема.

Теорема 5. *Пусть v_1 и v_2 — две вершины 5 степени графа krj. Тогда для $n \geqslant 1$*

$$e_{krj}(v_1, n) = \begin{cases} \frac{16n}{3}, n \equiv 0(\mathrm{mod} 3) \\ \frac{16n-1}{3}, n \equiv 1(\mathrm{mod} 3) \\ \frac{16n+1}{3}, n \equiv 2(\mathrm{mod} 3) \end{cases}, \quad e_{krj}(v_2, n) = \begin{cases} \frac{16n}{3}, n \equiv 0(\mathrm{mod} 6) \\ \frac{16n-1}{3}, n \equiv 1,4(\mathrm{mod} 6) \\ \frac{16n-2}{3}, n \equiv 2(\mathrm{mod} 6) \\ \frac{16n}{3} - 1, n \equiv 3(\mathrm{mod} 6) \\ \frac{16n+1}{3}, n \equiv 5(\mathrm{mod} 6) \end{cases}.$$

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On the manipulation of simple animations by dynamical distance geometry

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Аннотация

A simple approach for the manipulation of objects' animations, which is based on the definition and the solution of a *dynamical* Distance Geometry Problem (dynDGP), is discussed. An example in dimension 2 is presented where a video clip, designed to perform a psychological study in 1944, is manipulated by including new distance constraints. The obtained solution alters the perception of the "actions" performed in the original video clip.

1. Introduction

Given a positive integer $K > 0$ and a graph $G = (V \times T, E, d)$, the dynamical Distance Geometry Problem (dynDGP) [10] consists in finding a realization

$$x : (v, t) \in V \times T \longrightarrow x_v^t = x(v, t) \in \mathbb{R}^K$$

of G in the Euclidean space $\mathbb{R}^K$ such that the following objective is minimized:

$$\sigma(x) = \sum_{\substack{u,v \in V \\ t,q \in T}} \frac{|\,||x_u^q - x_v^t|| - d(u_q, v_t)\,|}{d(u_q, v_t)},$$

where $|\cdot|$ is the absolute value of a real number, and $||\cdot||$ is the Euclidean norm.

The dynamical component of a dynDGP (when compared to a standard Distance Geometry Problem (DGP)) is given by fact that the vertex set of the graph G is the set product between two sets. The set V contains "objects", whose nature strongly depends on the problem at hand. The second set T actually represents the time as a sequence of discrete steps. Notice that, from a formal point of view, the only difference between the DGP and the dynDGP is given by their corresponding vertex sets. However, the study and use of subgraphs of G with fixed times $t \in T$, as well as with fixed objects $v \in V$, can give important help in improving the performances of some solution methods that were initially designed for the DGP [7].

The function x represents animations as a set of trajectories $x(v, t)$, for every object $v \in V$, and for every time $t \in T$. A given function x may either be the representation of a known animation, or a possible realization of the graph G. In order to manipulate an existing animations x', a transformation of its representation in distance space in performed, so that the manipulation of the animation can subsequently be performed by introducing new distance constraints. The graph G is obtained therefore by computing a subset of necessary distances extracted from the existing animation x'. Then, new distance constraints are introduced in G, and a new animation x'', which is close as much as possible to x' while it also satisfies the new distance constraints, is generated by solving a dynDGP.

Since it is likely that the distance constraints generated from x' may not be fully compatible with the new included distance constraints, a realization x'' that satisfies the entire set of distances included in G may not exist. For this reason, in the context of the dynDGP, the realization that satisfies *as much as possible* all distance constraints is rather searched, i.e. a realization x'' that minimizes the function $\sigma(x'')$ but such that $\sigma(x'') > 0$ [3].

For this reason, in the context of the dynDGP, a second weight $\pi(u_q, v_t)$ is associated to every edge of G, which represents the *priority* of every distance $d(u_q, v_t)$. The function σ can be slightly modified so that every term of the sum is multiplied by the corresponding priority $\pi(u_q, v_t)$. In this way, a different importance is given to every distance constraint: when the original distance constraints (extracted from x') and new included constraints are not fully compatible, for example, one may want to give a higher priority to the new constraints, because they represent some features one wants to include in the resulting animations. For a larger discussion on this point, the reader is referred to [6].

There exist several methods for the solution of DGPs [5, 9]. The aim of this extended abstract is to show how manipulations on two-dimensional animations can be performed *via* dynDGP and by employing a method for local optimization for its solution. In fact, the constraints that are included in G for manipulating the animations are not supposed to drastically modify them, otherwise there would be no interest in using them as a base animation. Therefore, the original animations x' can be considered as good starting points for iterative methods for the solution of the dynDGP by local optimization. In the experiments reported below, a Spectral Projected Gradient (SPG) method is employed [1, 7].

2. Heider and Simmel animation

In 1944, Heider and Simmel published an original psychological study about perception, where panelists were asked to express the behavior they perceived while viewing a video clip depicting a set of animated objects. Such objects had, for the first time in this study, no human qualities, but they were rather represented with geometrical figures such as triangles (one larger and another smaller) and one circle, together with third geometrical figure representing the "house". The house was the only inanimate object, unless "pushed" by the other objects. The performed experiments showed that most people interpreted the movements of the geometrical objects as actions of animated beings (in most cases persons). Some panelists were even able to "see" short stories out of the object's animation [4].

The trajectories of the moving objects were extracted from the original video clip[2] (starting from frame 100). For every animated object, only the trajectory of their geometrical centers (center of the circle, and centers of the triangles) were included in x'; when creating the new video clip for the obtained animation x'', then, every object was represented again with its original geometrical figure. Notice, however, that these geometrical figures do not have any role in the determination of the solutions. The original animation x' is composed by 1800 frames; the "house" was not included in the experiments.

The dynDGP instance related to Heider and Simmel's video clip is generated with the idea of avoiding contacts among the objects "taking part" to the scene. Given the animation x' extracted from the video clip, the graph G is created by measuring some distances between the trajectories of the animated objects. Then, new distance constraints are included in G so that the distance between every pair of objects cannot be smaller than a given threshold. The environment where the scene takes place is represented by a box having sides 1 by 1; the additional constraints avoid that two objects get closer than the threshold $\Delta = 0.2$. For more details on how the graph G can be generated, the reader can refer to [10].

The video clip generated from the dynDGP solutions obtained with the SPG method are available online[3], together with other animations recently presented in other publications [7, 8]. In the particular context of the chosen animation, the new introduced distance constraints reflect the fact that the "characters" in the video should not come too close to each other. Starting from frame 250, the viewer can in fact see in the original animation x' that the two triangular objects are *attacking* each other by sudden and repetitive approaches. In the dynDGP-derived animation x'', instead, this approaching behavior is not pronounced, because of the imposed distance constraints. However, the attacking effect is a way amplified with this modification. In fact, in the animation x'', it is not necessary for one triangle to approach "too much" the other to make it step back; a much lighter approach is actually sufficient to have the stepping back effect over the other. Fig. 17 shows three key frames of the two animations x' and x''.

3. Conclusion

This extended abstract briefly discussed a simple method that, via the representation of an original animation in terms of distances and the solution of a dynDGP, allows to manipulate animations by imposing distance constraints. An animation that was initially used in a psychological study was presented and manipulated with new imposed constraints, which were able to alter the perception of the viewer about the "scene".

The dynDGP was recently introduced as a subclass of the DGP where solutions methods may be efficiently adapted to take advantage of the particular features of the problem [10]. Future works will

[2] `https://www.youtube.com/watch?v=sx7lBzHH7c8`

[3] `https://www.antoniomucherino.it/en/animations.php`

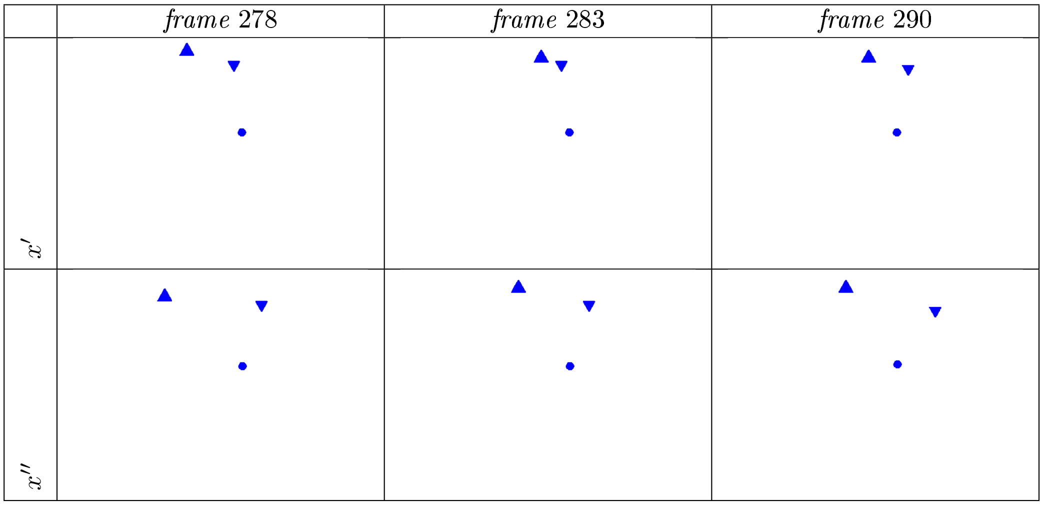

Рис. 17: *Comparisons between some key frames in the original animation and the obtained dynDGP solution. The new animation x'' is compared to the animation x' at frames 278 (the two triangles stare at each other), 283 (the big triangle attacks) and 290 (the small triangle makes a step back): the same behavior is visible in x'' but the two objects do not approach so much to each other.*

be focused on two particular points: (i) Can the dynDGP have benefits in the use of non-Euclidean distances [2], which might be able to better capture the dynamical component of these instances? (ii) Can other methods for optimization, or particularly designed for the DGP, be employed as more efficient alternatives to SPG?

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Сжатые измерения и равноугольные жесткие фреймы

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Compressed dimensions and equiangular hard frames

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Фреймом конечномерного гильбертова пространства называют набор векторов, линейная оболочка которых образует всё пространство. Понятие фрейма обобщает понятие базиса, так как в определении нет требования линейной независимости.

Набор векторов $\Phi = \{\varphi_n\}_{n=1}^N$ называется *фреймом* вещественного или комплексного пространства $\mathbb{H}^M$, если существуют константы $0 < A \le B < \infty$ такие что для всех $x \in \mathbb{H}^M$,

$$A\|x\|^2 \le \sum_{n=1}^{N} |\langle x, \varphi_n\rangle|^2 \le B\|x\|^2.$$

В конечномерном пространстве определение фрейма эквивалентно полноте системы векторов, то есть $\mathrm{span}\{\varphi_n\}_{n=1}^N = \mathbb{H}^M$.

Выделим следующие классы фреймов:

Φ — *жесткий фрейм,* если $A = B$.

Φ — *фрейм Парсеваля-Стеклова,* если $A = B = 1$.

Φ — *равномерный фрейм,* если $\|\varphi_{n'}\| = \|\varphi_{n''}\|$.

Φ — *равноугольный фрейм,* если Φ равномерный и существует $\beta \ge 0$ такое что $|\langle\varphi_{n'}, \varphi_{n''}\rangle| = \beta$ для всех $n' \ne n''$.

Φ — *равноугольный фрейм Парсеваля (EPF)*, если Φ равноугольный и фрейм Парсеваля - Стеклова.

Каждый фрейм определяет три линейных оператора. *Оператор анализа* — это оператор $V : \mathbb{H}^M \to \mathbb{H}^N$, определенный соотношениями $(Vx)_n = \langle x, \varphi_n\rangle,\ n = 1, \dots, N$.

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