deep learning, unsupervised learning and image retrieval

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Rennes, October 2017

image retrieval challenges

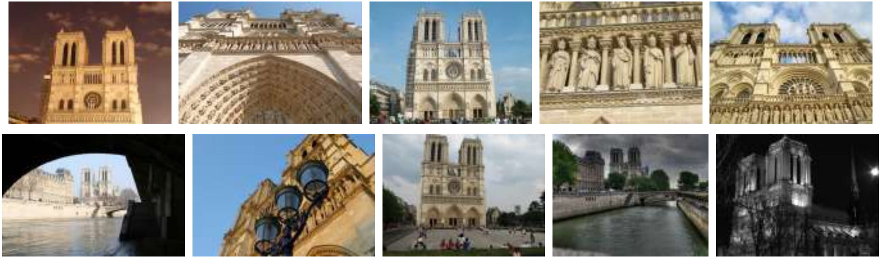
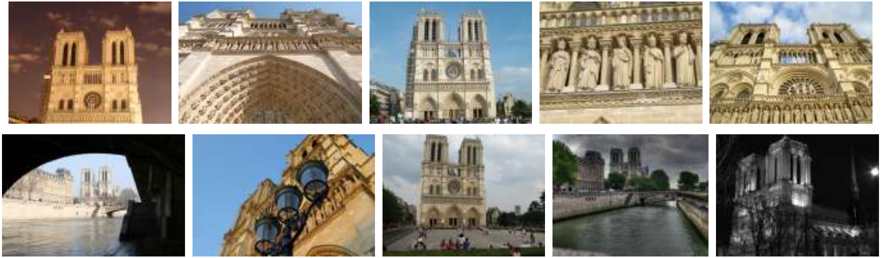


image retrieval challenges



- scale
- viewpoint
- occlusion
- clutter
- lighting
- distinctiveness
- distractors

image classification challenges

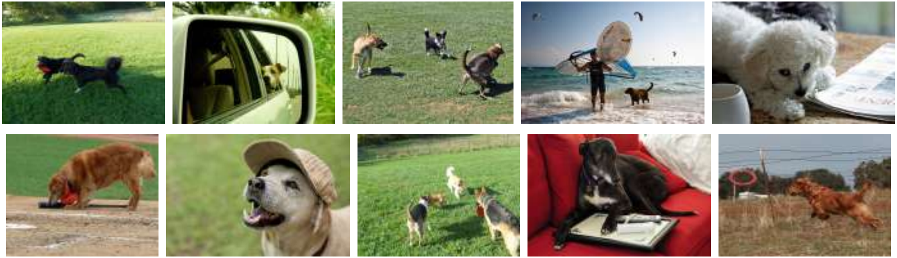









image classification challenges



- scale
- viewpoint
- occlusion
- clutter
- lighting
- number of instances
- texture/color
- pose
- deformability
- intra-class variability

data-driven approach

data-driven approach

feature extraction

representation

data-driven approach

feature extraction

representation

parameters → classifier

“dog”

“cat”

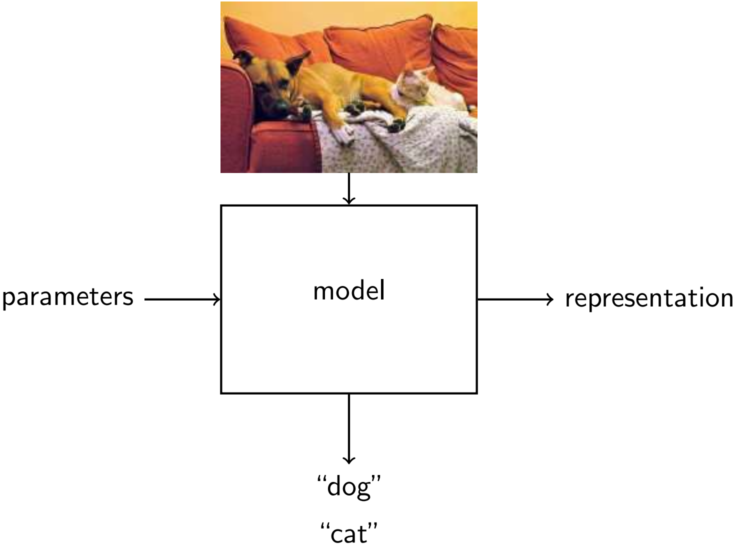
data-driven approach

parameters → model

"dog"

"cat"

data-driven approach



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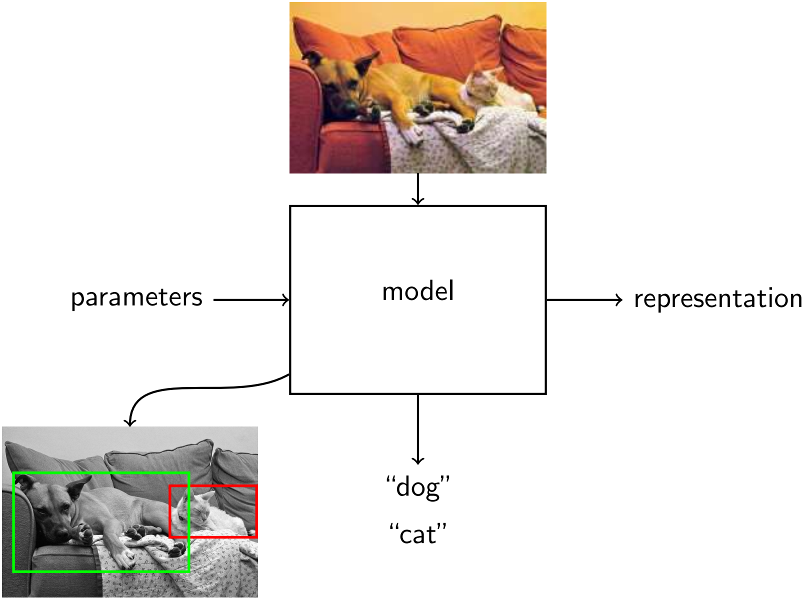

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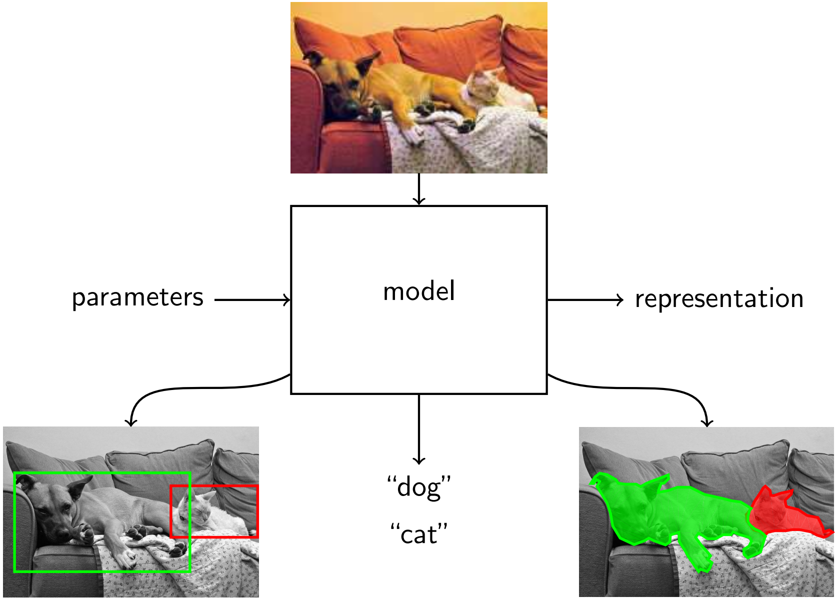

overview

- neural networks
- convolution
- image retrieval
- graph-based methods

neural networks

logistic regression

- class activations

$$a_k = \mathbf{w}_k^\top \mathbf{x} + b_k$$

- posterior class probabilities: softmax

$$y_k(\mathbf{x}) = \mathrm{softmax}_k(\mathbf{a}) := \frac{e^{a_k}}{\sum_j e^{a_j}}$$

logistic regression

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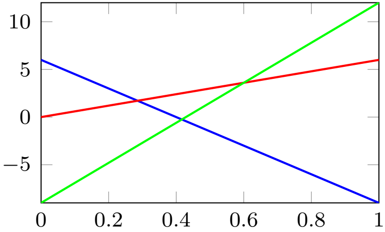

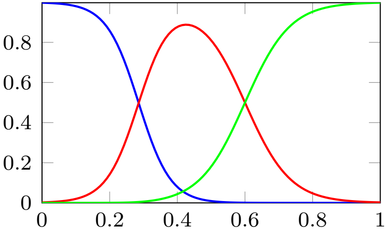

logistic regression

- class activations

$$a_k = \mathbf{w}_k^\top \mathbf{x} + b_k = \ln p(\mathbf{x}|\mathcal{C}_k)p(\mathcal{C}_k)$$

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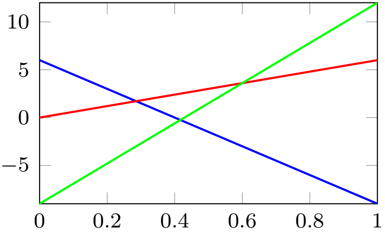
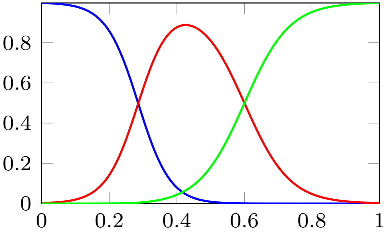




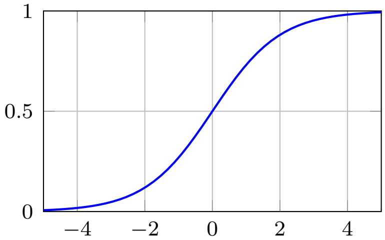
binary logistic regression

- activation

$$a = \mathbf{w}^\top \mathbf{x} + b$$

- posterior probability of class $\mathcal{C}_1$: sigmoid

$$y(\mathbf{x}) = \sigma(a) := \frac{1}{1 + e^{-a}}$$



binary logistic regression

- activation

$$a = \mathbf{w}^\top \mathbf{x} + b = \ln \frac{p(\mathbf{x}|\mathcal{C}_1)p(\mathcal{C}_1)}{p(\mathbf{x}|\mathcal{C}_2)p(\mathcal{C}_2)}$$

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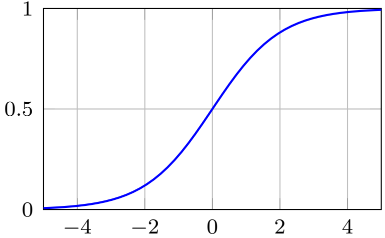

cross-entropy loss function

- input samples $\mathbf{X} = (x_{nd})$, activations $\mathbf{A} = (a_{nk})$
- output class probabilities $\mathbf{Y} = (y_{nk})$, $y_{nk} = \mathrm{softmax}_k(\mathbf{a}_n)$
- target variables $\mathbf{T} = (t_{nk})$, $t_{nk} = \mathbb{1}[\mathbf{x}_n \in \mathcal{C}_k]$
- average cross-entropy

$$L = -\ln p(\mathbf{T}) = -\frac{1}{N} \sum_n \sum_k t_{nk} \ln y_{nk}$$

- gradient

$$\frac{\partial L}{\partial \mathbf{A}} = \frac{1}{N}(\mathbf{Y} - \mathbf{T})$$

by increasing a class activation, the loss decreases if the class is correct, and increases otherwise

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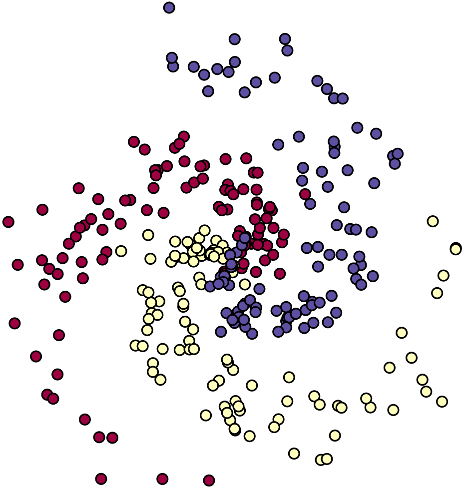
toy example



credit: Andrej Karpathy

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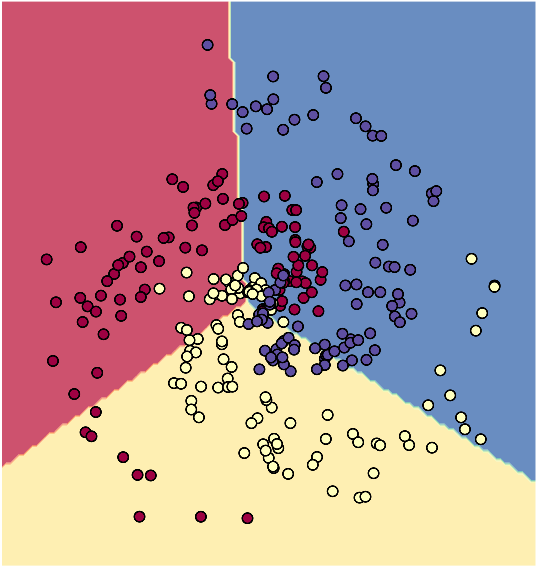

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two-layer network

- describe each sample with a feature vector obtained by a nonlinear function
- model this function after a (binary) logistic regression unit
- layer 1 activations → "features"

$$\mathbf{z} = h(\mathbf{W}_1^\top \mathbf{x} + \mathbf{b}_1)$$

- layer 2 activations → class probabilities

$$\mathbf{y} = \mathrm{softmax}(\mathbf{W}_2^\top \mathbf{z} + \mathbf{b}_2)$$

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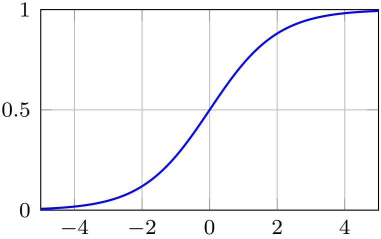
activation function h

sigmoid (element-wise)

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



rectified linear unit (ReLU)

$$\mathrm{relu}(x) = [x]_+ = \max(0, x)$$

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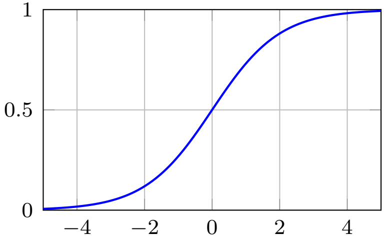

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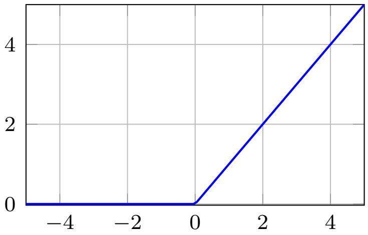

optimization

- input samples $\mathbf{X} = (x_{nd})$, output class probabilities $\mathbf{Y} = (y_{nk})$
- target variables $\mathbf{T} = (t_{nk})$
- network parameters $\boldsymbol{\theta} = ((\mathbf{W}_1, \mathbf{b}_1), (\mathbf{W}_2, \mathbf{b}_2))$
- loss function

$$L = f(\mathbf{X}, \mathbf{T}; \boldsymbol{\theta}) = -\frac{1}{N} \sum_n \sum_k t_{nk} \ln y_{nk} + \frac{\lambda}{2} (\|W_1\|_F^2 + \|W_2\|_F^2)$$

- optimization

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} f(\mathbf{X}, \mathbf{T}; \boldsymbol{\theta})$$

- gradient descent

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \epsilon \frac{\partial f}{\partial \boldsymbol{\theta}} (\mathbf{X}, \mathbf{T}; \boldsymbol{\theta}^t)$$

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data term

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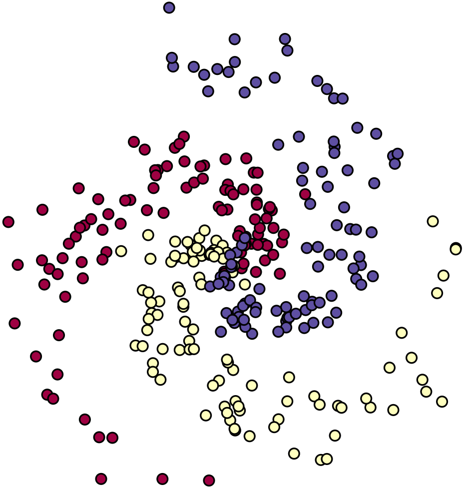

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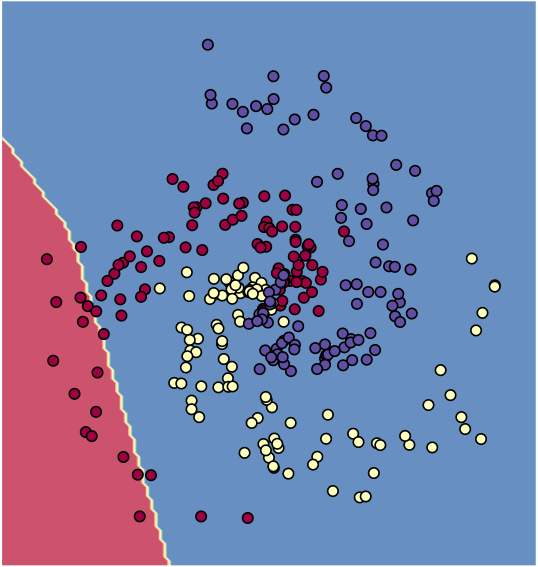

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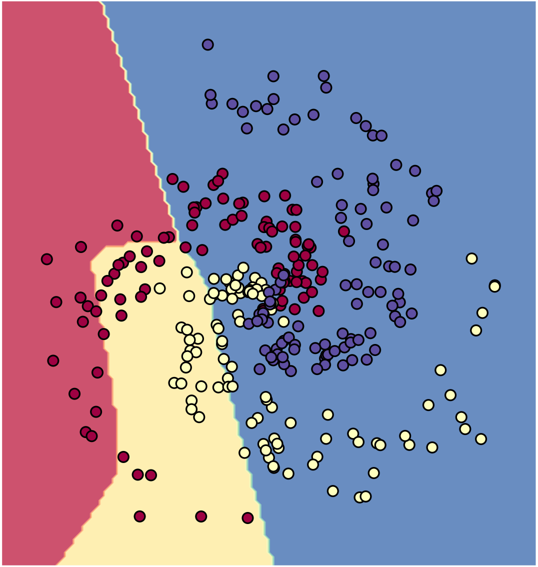

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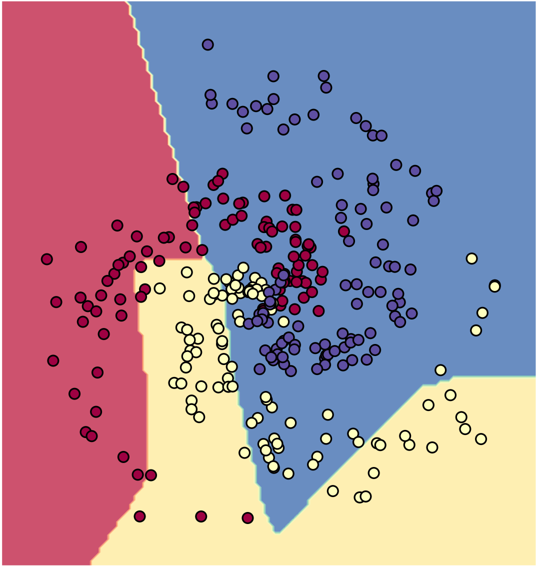

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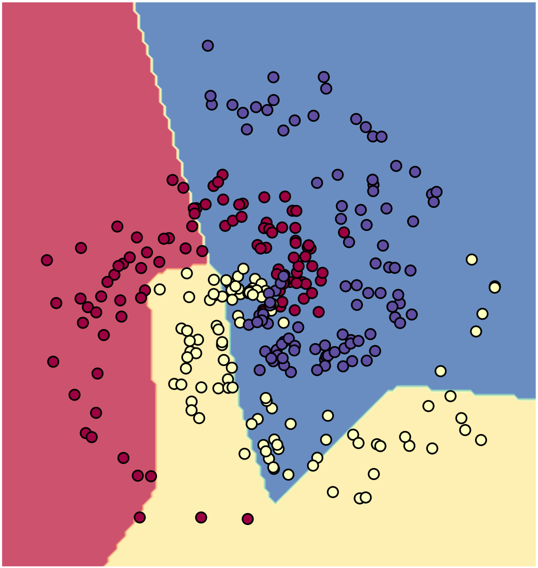

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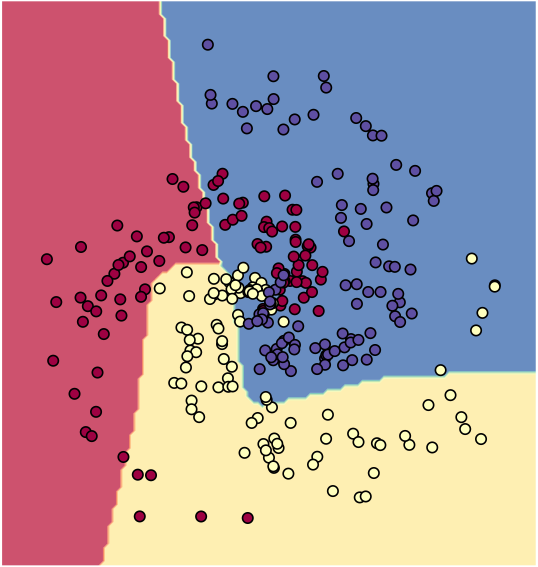

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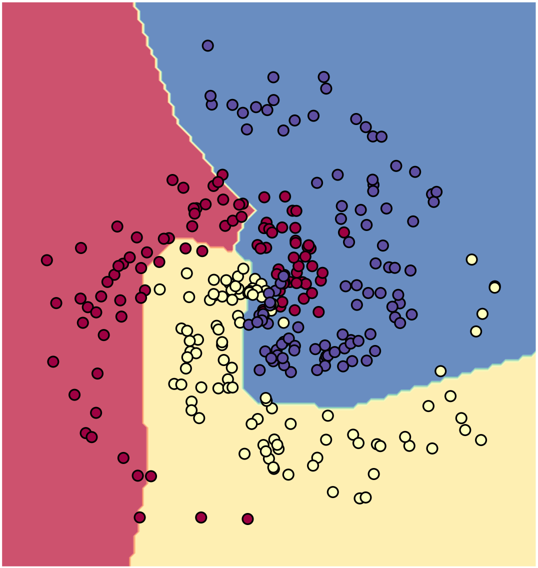

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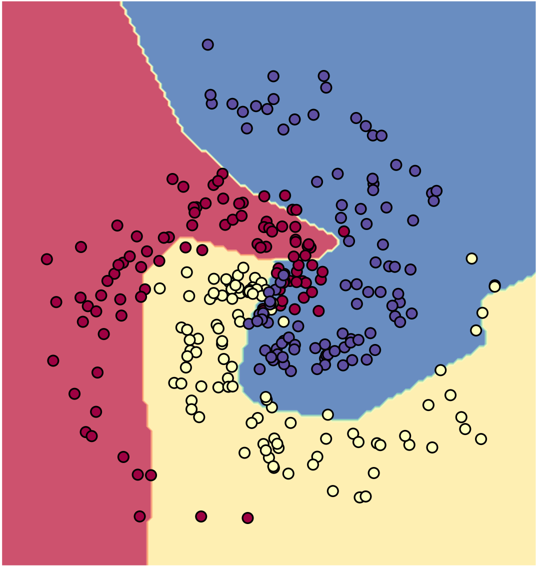

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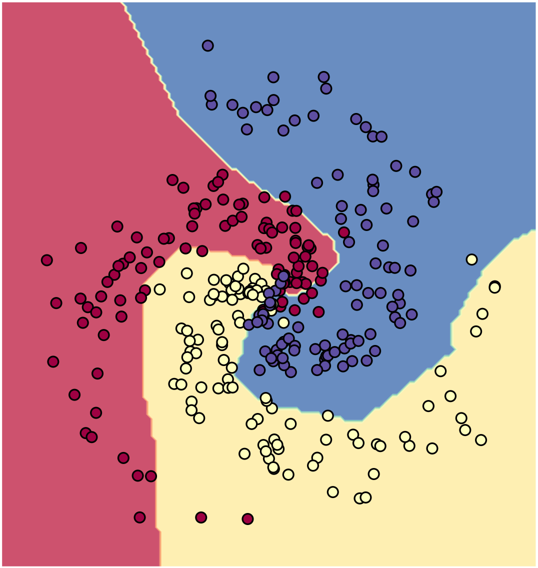

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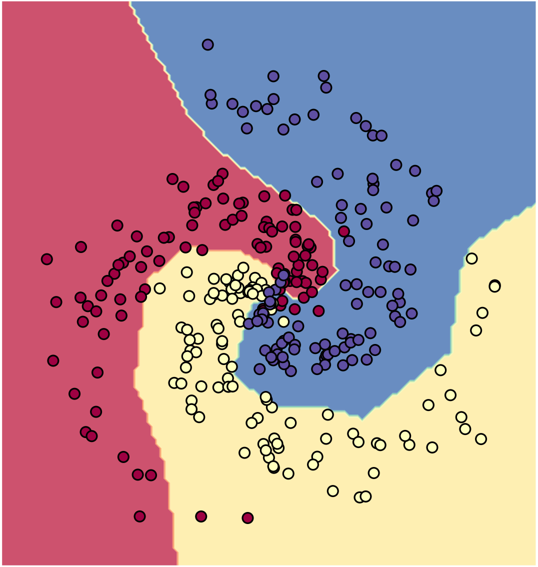

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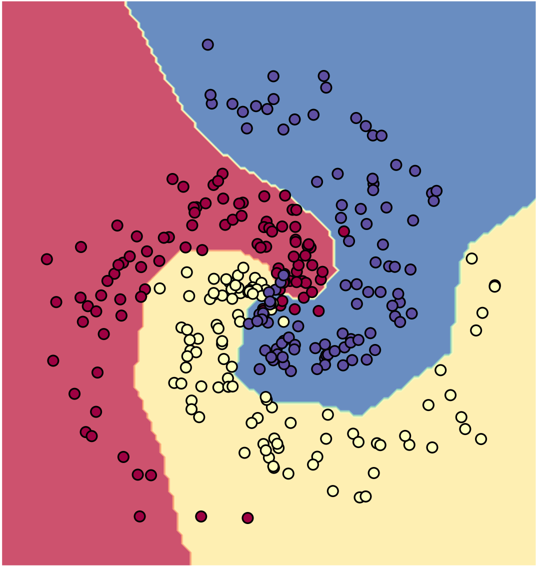

computing the gradient

- chain rule: if f is differentiable at $\mathbf{x}$ and g is differentiable at $\mathbf{y} = f(\mathbf{x})$, then $g \circ f$ is differentiable at $\mathbf{x}$ and

$$D(g \circ f)(\mathbf{x}) = Dg(\mathbf{y}) \cdot Df(\mathbf{x})$$

- how to use it:

$$\frac{\partial L}{\partial \mathbf{x}_1} = \frac{\partial L}{\partial \mathbf{x}_2} \cdot \frac{\partial \mathbf{x}_2}{\partial \mathbf{x}_1}$$

$$d\mathbf{x}_1 = d\mathbf{x}_2 \cdot Df(\mathbf{x}_1)$$

$$\mathbf{x} \to \mathbf{x}_1 \to \mathbf{x}_2 \to \mathbf{x}_3 \to \mathbf{x}_4 \to \mathbf{x}_5 \to L$$

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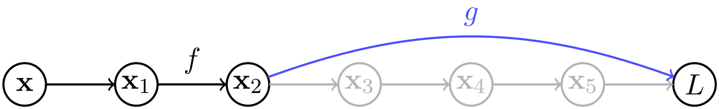

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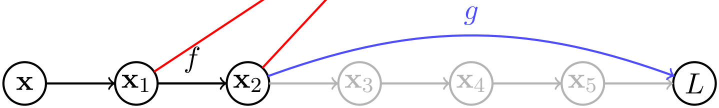

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$$d\mathbf{x}_1 = d\mathbf{x}_2 \cdot Df(\mathbf{x}_1)$$

$$\begin{tikzcd}
\mathbf{x} \arrow[r] & \mathbf{x}_1 \arrow[r, "f"] & \mathbf{x}_2 \arrow[r] \arrow[rrrr, bend left, "g"] & \mathbf{x}_3 \arrow[r] & \mathbf{x}_4 \arrow[r] & \mathbf{x}_5 \arrow[r] & L
\end{tikzcd}$$

computing the gradient

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$$\boxed{D(g \circ f)(\mathbf{x})} = Dg(\mathbf{y}) \cdot Df(\mathbf{x})$$

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$$\boxed{\frac{\partial L}{\partial \mathbf{x}_1}} = \frac{\partial L}{\partial \mathbf{x}_2} \cdot \frac{\partial \mathbf{x}_2}{\partial \mathbf{x}_1}$$

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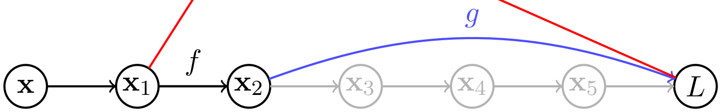

computing the gradient

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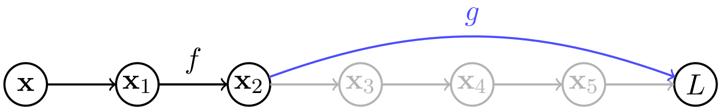
$$D(g \circ f)(\mathbf{x}) = Dg(\mathbf{y}) \cdot Df(\mathbf{x})$$

- how to use it:

$$\frac{\partial L}{\partial \mathbf{x}_1} = \frac{\partial L}{\partial \mathbf{x}_2} \cdot \frac{\partial \mathbf{x}_2}{\partial \mathbf{x}_1}$$

$$d\mathbf{x}_1 = d\mathbf{x}_2 \cdot Df(\mathbf{x}_1)$$



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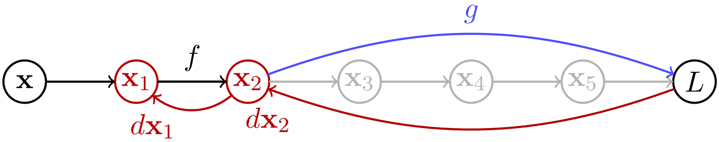
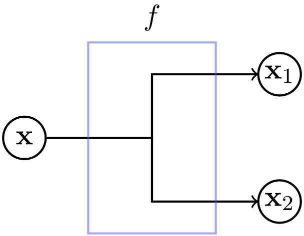
$$d\mathbf{x}_1 = d\mathbf{x}_2 \cdot Df(\mathbf{x}_1)$$



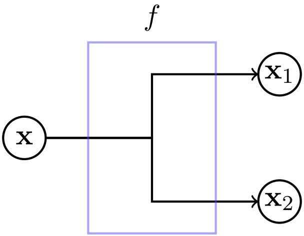
variable sharing



$$Df(\mathbf{x}) = \frac{\partial(\mathbf{x}_1, \mathbf{x}_2)}{\partial \mathbf{x}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{\partial}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial}{\partial \mathbf{x}_1} & \frac{\partial}{\partial \mathbf{x}_2} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{\partial}{\partial \mathbf{x}_1} + \frac{\partial}{\partial \mathbf{x}_2}$$

variable sharing



$$Df(\mathbf{x}) = \frac{\partial(\mathbf{x}_1, \mathbf{x}_2)}{\partial \mathbf{x}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

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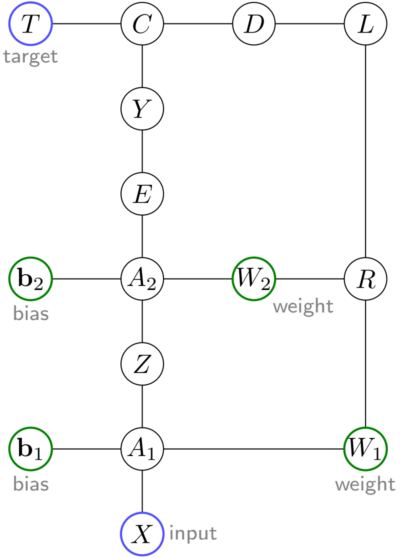
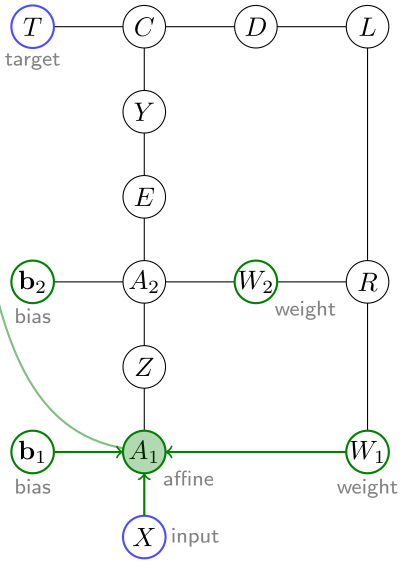
backpropagation



backpropagation

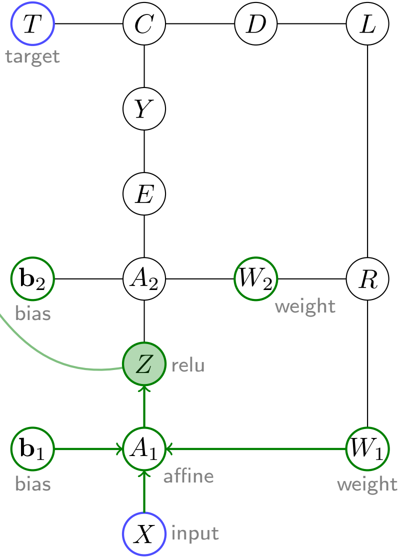
$$\boxed{A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1}$$



backpropagation

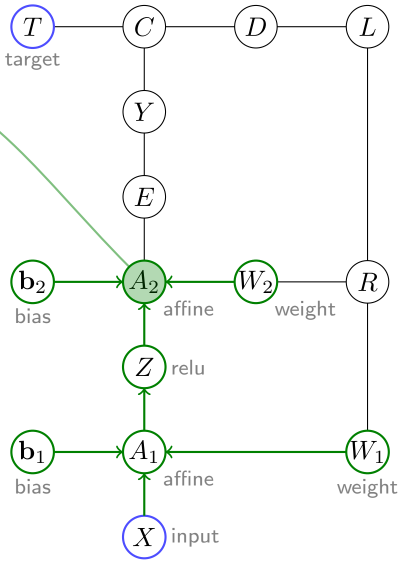
$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$

$$\boxed{Z = \textcolor{blue}{\max}(0, A_1)}$$



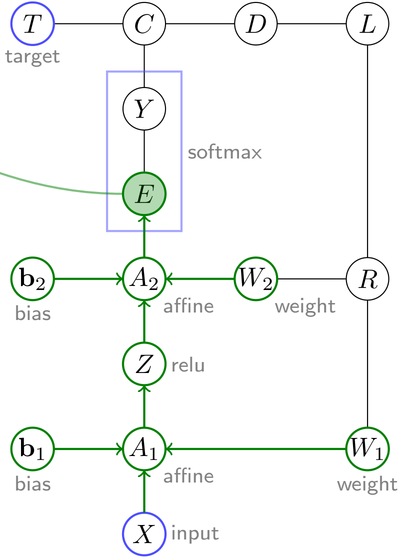
backpropagation

$$A_1 = \color{blue}{\mathrm{dot}}\color{black}(X, W_1) + \mathbf{b}_1$$
$$Z = \color{blue}{\max}\color{black}(0, A_1)$$
$$\boxed{A_2 = \color{blue}{\mathrm{dot}}\color{black}(Z, W_2) + \mathbf{b}_2}$$



backpropagation

$A_1 = \color{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$
$Z = \color{blue}{\max}(0, A_1)$
$A_2 = \color{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$
$\boxed{E = \color{blue}{\exp}(A_2)}$
$Y = E / \color{blue}{\mathrm{sum}_1}(E)$
$C = -\color{blue}{\mathrm{sum}_1}(T * \color{blue}{\log}(Y))$
$D = \color{blue}{\mathrm{sum}_0}(C)/N$



backpropagation

$$A_1 = \mathrm{dot}(X, W_1) + \mathbf{b}_1$$
$$Z = \max(0, A_1)$$
$$A_2 = \mathrm{dot}(Z, W_2) + \mathbf{b}_2$$
$$E = \exp(A_2)$$
$$\boxed{Y = E/\mathrm{sum}_1(E)}$$
$$C = -\mathrm{sum}_1(T * \log(Y))$$
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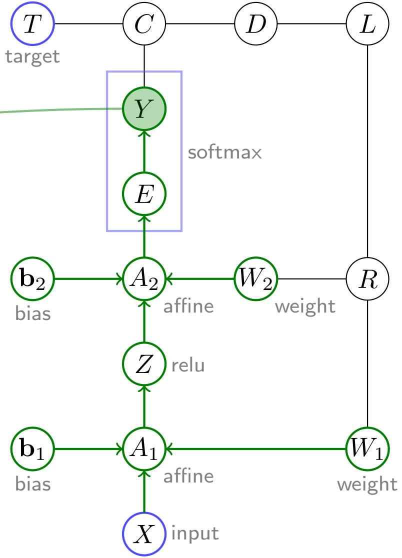
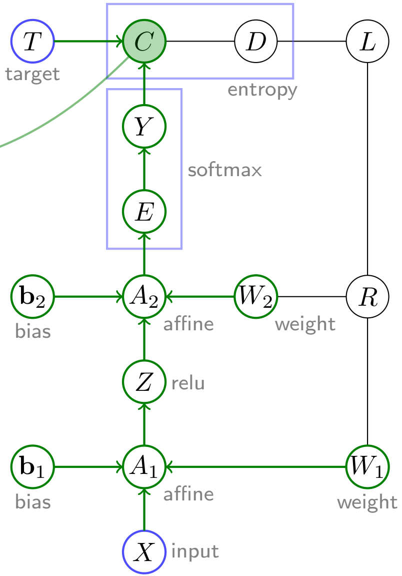


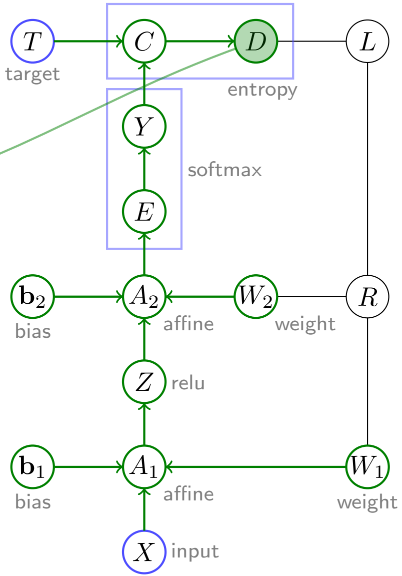
backpropagation

$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$
$$Z = \textcolor{blue}{\max}(0, A_1)$$
$$A_2 = \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$$
$$E = \textcolor{blue}{\exp}(A_2)$$
$$Y = E/\textcolor{blue}{\mathrm{sum}}_1(E)$$
$$\boxed{C = -\textcolor{blue}{\mathrm{sum}}_1(T * \textcolor{blue}{\log}(Y))}$$
$$D = \textcolor{blue}{\mathrm{sum}}_0(C)/N$$



backpropagation

$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$
$$Z = \textcolor{blue}{\max}(0, A_1)$$
$$A_2 = \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$$
$$E = \textcolor{blue}{\exp}(A_2)$$
$$Y = E/\textcolor{blue}{\mathrm{sum}}_1(E)$$
$$C = -\textcolor{blue}{\mathrm{sum}}_1(T * \textcolor{blue}{\log}(Y))$$
$$\boxed{D = \textcolor{blue}{\mathrm{sum}}_0(C)/N}$$



backpropagation

$$A_1 = \text{dot}(X, W_1) + \mathbf{b}_1$$
$$Z = \max(0, A_1)$$
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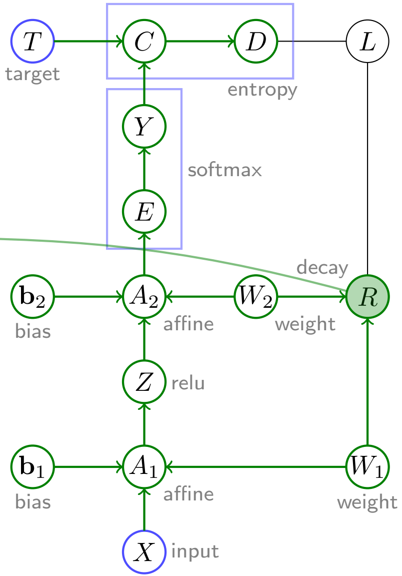
$$Y = E/\text{sum}_1(E)$$
$$C = -\text{sum}_1(T * \log(Y))$$
$$D = \text{sum}_0(C)/N$$
$$\boxed{R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)}$$



backpropagation

$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$
$$Z = \textcolor{blue}{\max}(0, A_1)$$
$$A_2 = \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$$
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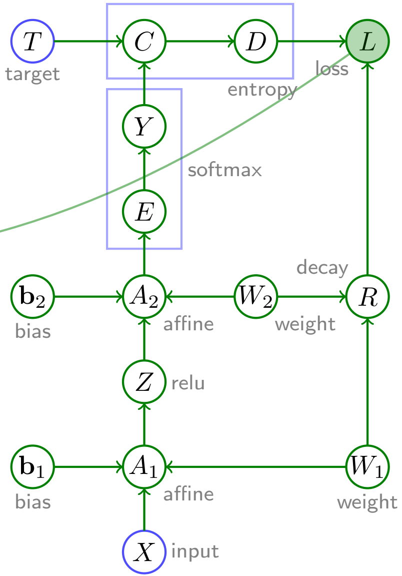
$$C = -\textcolor{blue}{\mathrm{sum}_1}(T * \textcolor{blue}{\log}(Y))$$
$$D = \textcolor{blue}{\mathrm{sum}_0}(C)/N$$
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$\boxed{L = D + R}$$



backpropagation

$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$
$Z = \textcolor{blue}{\max}(0, A_1)$
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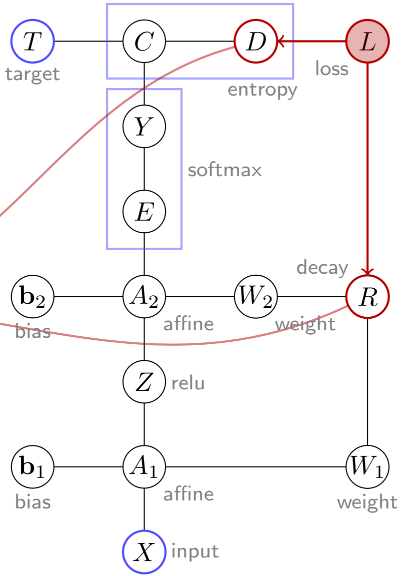
$C = -\textcolor{blue}{\mathrm{sum}_1}(T * \textcolor{blue}{\log}(Y))$
$D = \textcolor{blue}{\mathrm{sum}_0}(C)/N$
$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$
$L = D + R$
$\boxed{(dD, dR) = (dL, dL)}$



backpropagation

$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$
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$$A_2 = \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$$
$$E = \textcolor{blue}{\exp}(A_2)$$
$$Y = E / \textcolor{blue}{\mathrm{sum}_1}(E)$$
$$C = -\textcolor{blue}{\mathrm{sum}_1}(T * \textcolor{blue}{\log}(Y))$$
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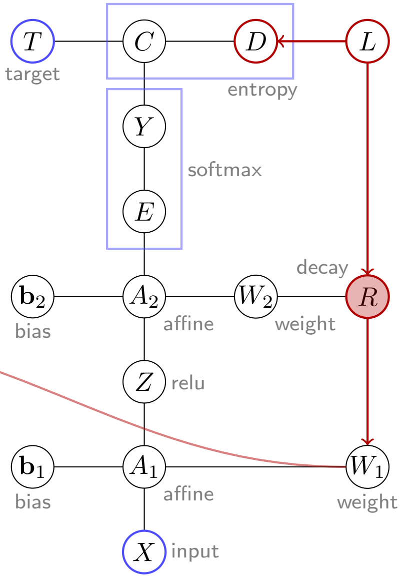
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$L = D + R$$
$$(dD, dR) = (dL, dL)$$
$$\boxed{dW_1 = dR * \lambda * W_1}$$
$$\boxed{dW_2 = dR * \lambda * W_2}$$



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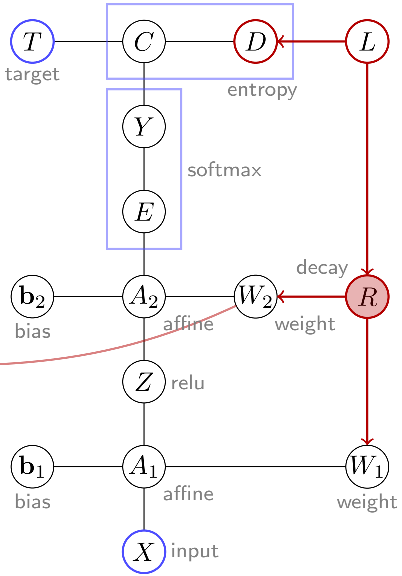

backpropagation

$$
\begin{aligned}
A_1 &= \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1 \\
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A_2 &= \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2 \\
E &= \textcolor{blue}{\exp}(A_2) \\
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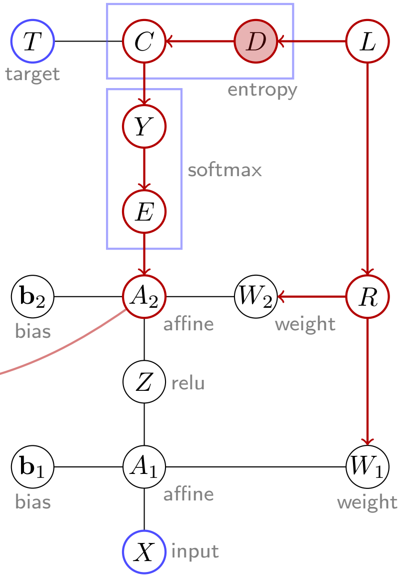
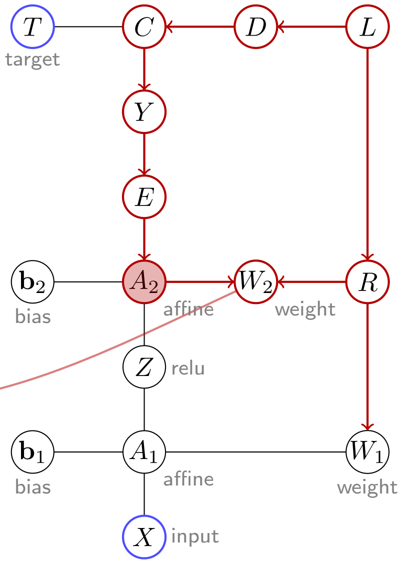
C &= -\textcolor{blue}{\mathrm{sum}_1}(T * \textcolor{blue}{\log}(Y)) \\
D &= \textcolor{blue}{\mathrm{sum}_0}(C)/N \\
R &= \tfrac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2) \\
L &= D + R \\
(dD, dR) &= (dL, dL) \\
dW_1 &= dR * \lambda * W_1 \\
dW_2 &= dR * \lambda * W_2 \\
&\boxed{dA_2 = dD * (Y - T)/N}
\end{aligned}
$$



backpropagation

$$A_1 = \mathrm{dot}(X, W_1) + \mathbf{b}_1$$
$$Z = \max(0, A_1)$$
$$A_2 = \mathrm{dot}(Z, W_2) + \mathbf{b}_2$$
$$E = \exp(A_2)$$
$$Y = E/\mathrm{sum}_1(E)$$
$$C = -\mathrm{sum}_1(T * \log(Y))$$
$$D = \mathrm{sum}_0(C)/N$$
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$L = D + R$$
$$(dD, dR) = (dL, dL)$$
$$dW_1 = dR * \lambda * W_1$$
$$dW_2 = dR * \lambda * W_2$$
$$dA_2 = dD * (Y - T)/N$$
$$\boxed{dW_2 \mathrel{+}= \mathrm{dot}(Z^\top, dA_2)}$$
$$d\mathbf{b}_2 = \mathrm{sum}_0(dA_2)$$
$$dZ = \mathrm{dot}(dA_2, W_2^\top)$$



backpropagation

$A_1 = \text{dot}(X, W_1) + \mathbf{b}_1$
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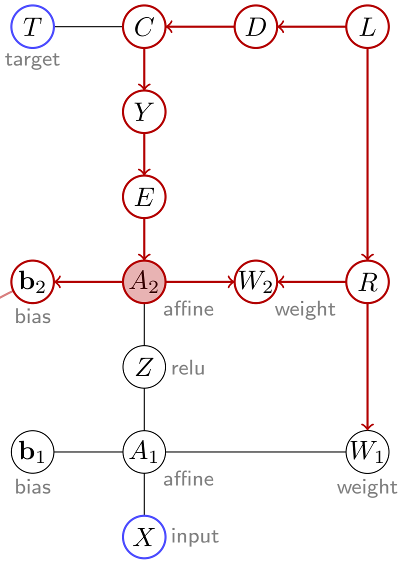
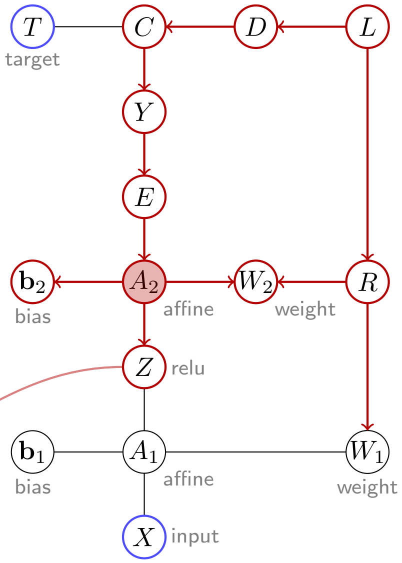
$dA_2 = dD * (Y - T)/N$
$dW_2 \mathrel{+}= \text{dot}(Z^\top, dA_2)$
$\boxed{d\mathbf{b}_2 = \text{sum}_0(dA_2)}$
$dZ = \text{dot}(dA_2, W_2^\top)$



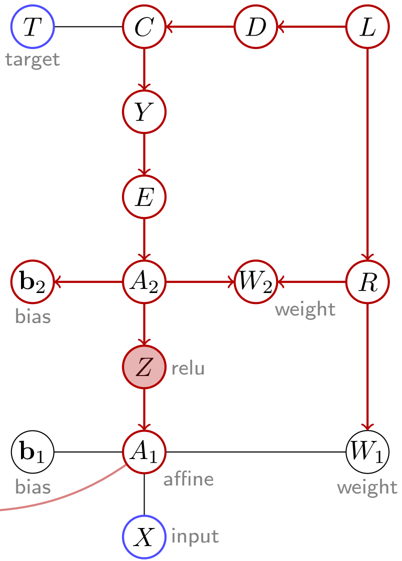
backpropagation

$$
\begin{aligned}
A_1 &= \text{dot}(X, W_1) + \mathbf{b}_1 \\
Z &= \max(0, A_1) \\
A_2 &= \text{dot}(Z, W_2) + \mathbf{b}_2 \\
E &= \exp(A_2) \\
Y &= E/\text{sum}_1(E) \\
C &= -\text{sum}_1(T * \log(Y)) \\
D &= \text{sum}_0(C)/N \\
R &= \tfrac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2) \\
L &= D + R \\
(dD, dR) &= (dL, dL) \\
dW_1 &= dR * \lambda * W_1 \\
dW_2 &= dR * \lambda * W_2 \\
dA_2 &= dD * (Y - T)/N \\
dW_2 &\mathrel{+}= \text{dot}(Z^\top, dA_2) \\
d\mathbf{b}_2 &= \text{sum}_0(dA_2) \\
&\boxed{dZ = \text{dot}(dA_2, W_2^\top)}
\end{aligned}
$$



backpropagation

$A_1 = \text{dot}(X, W_1) + \mathbf{b}_1$
$Z = \max(0, A_1)$
$A_2 = \text{dot}(Z, W_2) + \mathbf{b}_2$
$E = \exp(A_2)$
$Y = E/\text{sum}_1(E)$
$C = -\text{sum}_1(T * \log(Y))$
$D = \text{sum}_0(C)/N$
$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$
$L = D + R$
$(dD, dR) = (dL, dL)$
$dW_1 = dR * \lambda * W_1$
$dW_2 = dR * \lambda * W_2$
$dA_2 = dD * (Y - T)/N$
$dW_2 \mathrel{+}= \text{dot}(Z^\top, dA_2)$
$d\mathbf{b}_2 = \text{sum}_0(dA_2)$
$dZ = \text{dot}(dA_2, W_2^\top)$
$\boxed{dA_1 = dZ * (Z > 0)}$

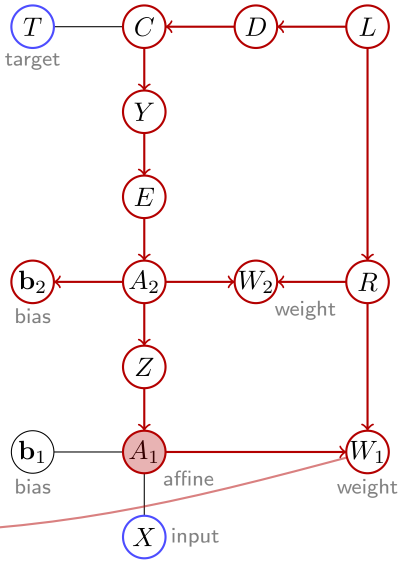


backpropagation

$$
\begin{aligned}
A_1 &= \mathrm{dot}(X, W_1) + \mathbf{b}_1 \\
Z &= \max(0, A_1) \\
A_2 &= \mathrm{dot}(Z, W_2) + \mathbf{b}_2 \\
E &= \exp(A_2) \\
Y &= E / \mathrm{sum}_1(E) \\
C &= -\mathrm{sum}_1(T * \log(Y)) \\
D &= \mathrm{sum}_0(C)/N \\
R &= \tfrac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2) \\
L &= D + R \\
(dD, dR) &= (dL, dL) \\
dW_1 &= dR * \lambda * W_1 \\
dW_2 &= dR * \lambda * W_2 \\
dA_2 &= dD * (Y - T)/N \\
dW_2 &\mathrel{+}= \mathrm{dot}(Z^\top, dA_2) \\
d\mathbf{b}_2 &= \mathrm{sum}_0(dA_2) \\
dZ &= \mathrm{dot}(dA_2, W_2^\top) \\
dA_1 &= dZ * (Z > 0)
\end{aligned}
$$

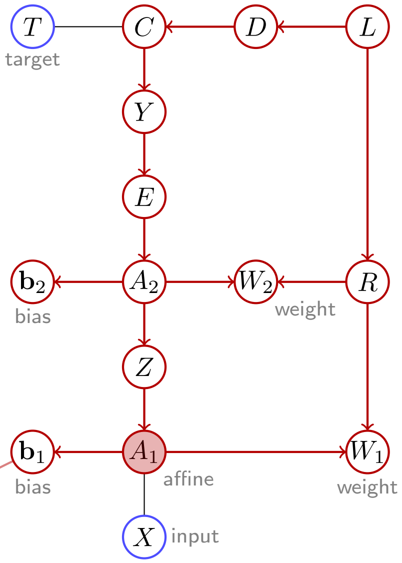
$$\boxed{dW_1 \mathrel{+}= \mathrm{dot}(X^\top, dA_1)}$$

$$\boxed{d\mathbf{b}_1 = \mathrm{sum}_0(dA_1)}$$



backpropagation

$$
\begin{aligned}
A_1 &= \mathrm{dot}(X, W_1) + \mathbf{b}_1 \\
Z &= \max(0, A_1) \\
A_2 &= \mathrm{dot}(Z, W_2) + \mathbf{b}_2 \\
E &= \exp(A_2) \\
Y &= E / \mathrm{sum}_1(E) \\
C &= -\mathrm{sum}_1(T * \log(Y)) \\
D &= \mathrm{sum}_0(C)/N \\
R &= \tfrac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2) \\
L &= D + R \\
(dD, dR) &= (dL, dL) \\
dW_1 &= dR * \lambda * W_1 \\
dW_2 &= dR * \lambda * W_2 \\
dA_2 &= dD * (Y - T)/N \\
dW_2 &\mathrel{+}= \mathrm{dot}(Z^\top, dA_2) \\
d\mathbf{b}_2 &= \mathrm{sum}_0(dA_2) \\
dZ &= \mathrm{dot}(dA_2, W_2^\top) \\
dA_1 &= dZ * (Z > 0) \\
dW_1 &\mathrel{+}= \mathrm{dot}(X^\top, dA_1) \\
d\mathbf{b}_1 &= \mathrm{sum}_0(dA_1)
\end{aligned}
$$



automatic differentiation

$$
\begin{aligned}
A_1 &= \mathrm{dot}(X, W_1) + \mathbf{b}_1 \\
Z &= \max(0, A_1) \\
A_2 &= \mathrm{dot}(Z, W_2) + \mathbf{b}_2 \\
E &= \exp(A_2) \\
Y &= E / \mathrm{sum}_1(E) \\
C &= -\mathrm{sum}_1(T * \log(Y)) \\
D &= \mathrm{sum}_0(C)/N \\
R &= \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2) \\
L &= D + R
\end{aligned}
$$

$$
\begin{aligned}
(dD, dR) &= (dL, dL) \\
dW_1 &= dR * \lambda * W_1 \\
dW_2 &= dR * \lambda * W_2 \\
dA_2 &= dD * (Y - T)/N \\
dW_2 &\mathrel{+}= \mathrm{dot}(Z^\top, dA_2) \\
d\mathbf{b}_2 &= \mathrm{sum}_0(dA_2) \\
dZ &= \mathrm{dot}(dA_2, W_2^\top) \\
dA_1 &= dZ * (Z > 0) \\
dW_1 &\mathrel{+}= \mathrm{dot}(X^\top, dA_1) \\
d\mathbf{b}_1 &= \mathrm{sum}_0(dA_1)
\end{aligned}
$$

what is an easy way to automatically generate the backward code from the forward one?

automatic differentiation

$$A_1 = \textcolor{blue}{\mathrm{dot}}(X, W_1) + \mathbf{b}_1$$
$$\boxed{Z = \textcolor{blue}{\max}(0, A_1)}$$
$$A_2 = \textcolor{blue}{\mathrm{dot}}(Z, W_2) + \mathbf{b}_2$$
$$E = \textcolor{blue}{\exp}(A_2)$$
$$Y = E/\textcolor{blue}{\mathrm{sum}}_1(E)$$
$$C = -\textcolor{blue}{\mathrm{sum}}_1(T * \textcolor{blue}{\log}(Y))$$
$$D = \textcolor{blue}{\mathrm{sum}}_0(C)/N$$
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$L = D + R$$
$$(dD, dR) = (dL, dL)$$
$$dW_1 = dR * \lambda * W_1$$
$$dW_2 = dR * \lambda * W_2$$
$$dA_2 = dD * (Y - T)/N$$
$$dW_2 \mathrel{+}= \textcolor{blue}{\mathrm{dot}}(Z^\top, dA_2)$$
$$d\mathbf{b}_2 = \textcolor{blue}{\mathrm{sum}}_0(dA_2)$$
$$dZ = \textcolor{blue}{\mathrm{dot}}(dA_2, W_2^\top)$$
$$\boxed{dA_1 = dZ * (Z > 0)}$$
$$dW_1 \mathrel{+}= \textcolor{blue}{\mathrm{dot}}(X^\top, dA_1)$$
$$d\mathbf{b}_1 = \textcolor{blue}{\mathrm{sum}}_0(dA_1)$$

def relu(A):
  $\boxed{Z = \textcolor{blue}{\max}(0, A)}$
  def back(dZ, dA):
    $\boxed{dA \mathrel{+}= dZ * (Z > 0)}$
  return $\mathit{node}(Z, \mathrm{back})$

automatic differentiation

$$A_1 = \mathrm{dot}(X, W_1) + \mathbf{b}_1$$
$$\boxed{Z = \mathrm{relu}(A_1)}$$
$$A_2 = \mathrm{dot}(Z, W_2) + \mathbf{b}_2$$
$$E = \exp(A_2)$$
$$Y = E/\mathrm{sum}_1(E)$$
$$C = -\mathrm{sum}_1(T * \log(Y))$$
$$D = \mathrm{sum}_0(C)/N$$
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$L = D + R$$
$$(dD, dR) = (dL, dL)$$
$$dW_1 = dR * \lambda * W_1$$
$$dW_2 = dR * \lambda * W_2$$
$$dA_2 = dD * (Y - T)/N$$
$$dW_2 \mathrel{+}= \mathrm{dot}(Z^\top, dA_2)$$
$$d\mathbf{b}_2 = \mathrm{sum}_0(dA_2)$$
$$dZ = \mathrm{dot}(dA_2, W_2^\top)$$
$$\boxed{Z.\mathrm{back}(A_1)}$$
$$dW_1 \mathrel{+}= \mathrm{dot}(X^\top, dA_1)$$
$$d\mathbf{b}_1 = \mathrm{sum}_0(dA_1)$$

def relu(A):
  $Z = \max(0, A)$
  def back(dZ, dA):
    $dA \mathrel{+}= dZ * (Z > 0)$
  return $node(Z, \mathrm{back})$

automatic differentiation

$$\boxed{A_1 = \mathrm{dot}(X, W_1) + \mathbf{b}_1}$$
$$Z = \mathrm{relu}(A_1)$$
$$\boxed{A_2 = \mathrm{dot}(Z, W_2) + \mathbf{b}_2}$$
$$E = \exp(A_2)$$
$$Y = E / \mathrm{sum}_1(E)$$
$$C = -\mathrm{sum}_1(T * \log(Y))$$
$$D = \mathrm{sum}_0(C)/N$$
$$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$$
$$L = D + R$$
$$(dD, dR) = (dL, dL)$$
$$dW_1 = dR * \lambda * W_1$$
$$dW_2 = dR * \lambda * W_2$$
$$dA_2 = dD * (Y - T)/N$$
$$\boxed{\begin{aligned} &dW_2 \mathrel{+}= \mathrm{dot}(Z^\top, dA_2) \\ &d\mathbf{b}_2 = \mathrm{sum}_0(dA_2) \\ &dZ = \mathrm{dot}(dA_2, W_2^\top) \end{aligned}}$$
$$Z.\mathrm{back}(A_1)$$
$$\boxed{\begin{aligned} &dW_1 \mathrel{+}= \mathrm{dot}(X^\top, dA_1) \\ &d\mathbf{b}_1 = \mathrm{sum}_0(dA_1) \end{aligned}}$$

def affine$(X, (W, \mathbf{b}))$:

$\quad\boxed{A = \mathrm{dot}(X, W) + \mathbf{b}}$

$\quad$**def** back$(dA, dX, (dW, d\mathbf{b}))$:

$$\boxed{\begin{aligned} &dW \mathrel{+}= \mathrm{dot}(X^\top, dA) \\ &d\mathbf{b} \mathrel{+}= \mathrm{sum}_0(dA) \\ &dX \mathrel{+}= \mathrm{dot}(dA, W^\top) \end{aligned}}$$

$\quad$**return** $\mathit{node}(A, \mathrm{back})$

automatic differentiation

$\boxed{A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))}$
$Z = \text{relu}(A_1)$
$\boxed{A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))}$
$E = \exp(A_2)$
$Y = E / \text{sum}_1(E)$
$C = -\text{sum}_1(T * \log(Y))$
$D = \text{sum}_0(C)/N$
$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$
$L = D + R$
$(dD, dR) = (dL, dL)$
$dW_1 = dR * \lambda * W_1$
$dW_2 = dR * \lambda * W_2$
$dA_2 = dD * (Y - T)/N$
$\boxed{A_2.\,\text{back}(Z, (W_2, \mathbf{b}_2))}$

$Z.\,\text{back}(A_1)$
$\boxed{A_1.\,\text{back}(X, (W_1, \mathbf{b}_1))}$

def affine$(X, (W, \mathbf{b}))$:
  $A = \text{dot}(X, W) + \mathbf{b}$
  def back$(dA, dX, (dW, d\mathbf{b}))$:
    $dW \mathrel{+}= \text{dot}(X^\top, dA)$
    $d\mathbf{b} \mathrel{+}= \text{sum}_0(dA)$
    $dX \mathrel{+}= \text{dot}(dA, W^\top)$
  return $\mathit{node}(A, \text{back})$

automatic differentiation

$A_1 = \mathrm{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \mathrm{relu}(A_1)$
$A_2 = \mathrm{affine}(Z, (W_2, \mathbf{b}_2))$
$E = \exp(A_2)$
$Y = E / \mathrm{sum}_1(E)$
$C = -\mathrm{sum}_1(T * \log(Y))$
$D = \mathrm{sum}_0(C)/N$
$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$
$L = D + R$
$(dD, dR) = (dL, dL)$
$dW_1 = dR * \lambda * W_1$
$dW_2 = dR * \lambda * W_2$
$dA_2 = dD * (Y - T)/N$
$A_2.\mathrm{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\mathrm{back}(A_1)$
$A_1.\mathrm{back}(X, (W_1, \mathbf{b}_1))$

def $\mathrm{entropy}(A, T)$:
 $E = \exp(A)$
 $Y = E / \mathrm{sum}_1(E)$
 $C = -\mathrm{sum}_1(T * \log(Y))$
 $D = \mathrm{sum}_0(C)/N$
 def $\mathrm{back}(dD, dA, \_)$:
  $dA \mathrel{+}= dD * (Y - T)/N$
 return $node(D, \mathrm{back})$

automatic differentiation

$A_1 = \mathrm{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \mathrm{relu}(A_1)$
$A_2 = \mathrm{affine}(Z, (W_2, \mathbf{b}_2))$
$\boxed{D = \mathrm{entropy}(A_2, T)}$

$R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)$
$L = D + R$
$(dD, dR) = (dL, dL)$
$dW_1 = dR * \lambda * W_1$
$dW_2 = dR * \lambda * W_2$
$\boxed{D.\,\mathrm{back}(A_2, T)}$
$A_2.\,\mathrm{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\,\mathrm{back}(A_1)$
$A_1.\,\mathrm{back}(X, (W_1, \mathbf{b}_1))$

def $\mathrm{entropy}(A, T)$:
    $E = \mathrm{exp}(A)$
    $Y = E/\mathrm{sum}_1(E)$
    $C = -\mathrm{sum}_1(T * \log(Y))$
    $D = \mathrm{sum}_0(C)/N$
    def $\mathrm{back}(dD, dA, \_)$:
        $dA \mathrel{+}= dD * (Y - T)/N$
    return $node(D, \mathrm{back})$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$D = \text{entropy}(A_2, T)$

$\boxed{R = \frac{\lambda}{2} * (\|W_1\|_F^2 + \|W_2\|_F^2)}$
$L = D + R$
$(dD, dR) = (dL, dL)$
$dW_1 = dR * \lambda * W_1$
$dW_2 = dR * \lambda * W_2$
$D.\,\text{back}(A_2, T)$
$A_2.\,\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\,\text{back}(A_1)$
$A_1.\,\text{back}(X, (W_1, \mathbf{b}_1))$

def $\text{decay}(W)$:
  $\boxed{R = \frac{\lambda}{2} * \text{sum}(\|w\|_F^2 \textbf{ for } w \textbf{ in } W)}$
  def $\text{back}(dR, dW)$:
    for (w, dw) **in** $\text{zip}(W, dW)$:
      $dw \mathrel{+}= dR * \lambda * w$
  return $\mathit{node}(R, \text{back})$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$D = \text{entropy}(A_2, T)$

$\boxed{R = \text{decay}((W_1, W_2))}$
$L = D + R$
$(dD, dR) = (dL, dL)$
$\boxed{R.\,\text{back}((W_1, W_2))}$

$D.\,\text{back}(A_2, T)$
$A_2.\,\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\,\text{back}(A_1)$
$A_1.\,\text{back}(X, (W_1, \mathbf{b}_1))$

def $\text{decay}(W)$:
  $R = \frac{\lambda}{2} * \text{sum}(\|w\|_F^2$ **for** w **in** $W)$
  def $\text{back}(dR, dW)$:
    for (w, dw) **in** $\text{zip}(W, dW)$:
      $dw \mathrel{+}= dR * \lambda * w$
  return $node(R, \text{back})$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$D = \text{entropy}(A_2, T)$

$R = \text{decay}((W_1, W_2))$
$\boxed{L = D + R}$
$\boxed{(dD, dR) = (dL, dL)}$
$R.\text{back}((W_1, W_2))$

$D.\text{back}(A_2, T)$
$A_2.\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\text{back}(A_1)$
$A_1.\text{back}(X, (W_1, \mathbf{b}_1))$

def $\text{add}(X)$:
  $\boxed{S = \text{sum}(X)}$
  def $\text{back}(dS, dX)$:
    for dx **in** dX:
      $dx \mathrel{+}= dS$
  return $\mathit{node}(S, \text{back})$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$D = \text{entropy}(A_2, T)$

$R = \text{decay}((W_1, W_2))$
$\boxed{L = \text{add}((D, R))}$
$\boxed{L.\,\text{back}((D, R))}$
$R.\,\text{back}((W_1, W_2))$

$D.\,\text{back}(A_2, T)$
$A_2.\,\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\,\text{back}(A_1)$
$A_1.\,\text{back}(X, (W_1, \mathbf{b}_1))$

```
def add(X):
    S = sum(X)
    def back(dS, dX):
        for dx in dX:
            dx += dS
    return node(S, back)
```

automatic differentiation

$A_1 = \mathrm{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \mathrm{relu}(A_1)$
$A_2 = \mathrm{affine}(Z, (W_2, \mathbf{b}_2))$
$D = \mathrm{entropy}(A_2, T)$

$R = \mathrm{decay}((W_1, W_2))$
$L = \mathrm{add}((D, R))$
$L.\mathrm{back}((D, R))$
$R.\mathrm{back}((W_1, W_2))$

$D.\mathrm{back}(A_2, T)$
$A_2.\mathrm{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\mathrm{back}(A_1)$
$A_1.\mathrm{back}(X, (W_1, \mathbf{b}_1))$

def $\mathrm{loss}(A, T, W)$:
    $D = \mathrm{entropy}(A, T)$
    $R = \mathrm{decay}(W)$
    $L = \mathrm{add}((D, R))$
    def $\mathrm{back}(A, T, W)$:
        $L.\mathrm{back}((D, R))$
        $R.\mathrm{back}(W)$
        $D.\mathrm{back}(A, T)$
    return $\mathit{block}(L, \mathrm{back})$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$\boxed{L = \text{loss}(A_2, T, (W_1, W_2))}$

def $\text{loss}(A, T, W)$:
  $D = \text{entropy}(A, T)$
  $R = \text{decay}(W)$
  $L = \text{add}((D, R))$
  def $\text{back}(A, T, W)$:
    $L.\,\text{back}((D, R))$
    $R.\,\text{back}(W)$
    $D.\,\text{back}(A, T)$
  return $\mathit{block}(L, \text{back})$

$\boxed{L.\,\text{back}(A_2, T, (W_1, W_2))}$

$A_2.\,\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\,\text{back}(A_1)$
$A_1.\,\text{back}(X, (W_1, \mathbf{b}_1))$

automatic differentiation

$A_1 = \text{affine}(X, (W_1, \mathbf{b}_1))$
$Z = \text{relu}(A_1)$
$A_2 = \text{affine}(Z, (W_2, \mathbf{b}_2))$
$L = \text{loss}(A_2, T, (W_1, W_2))$

def $\text{loss}(A, T, W)$:
    $L = \text{entropy}(A, T) + \text{decay}(W)$
    return $\mathit{block}(L)$

def $\text{loss}(A, T, W)$:
    $D = \text{entropy}(A, T)$
    $R = \text{decay}(W)$
    $L = \text{add}((D, R))$
    def $\text{back}(A, T, W)$:
        $L.\text{back}((D, R))$
        $R.\text{back}(W)$
        $D.\text{back}(A, T)$
    return $\mathit{block}(L, \text{back})$

$L.\text{back}(A_2, T, (W_1, W_2))$

$A_2.\text{back}(Z, (W_2, \mathbf{b}_2))$

$Z.\text{back}(A_1)$
$A_1.\text{back}(X, (W_1, \mathbf{b}_1))$

automatic differentiation

$$\boxed{\begin{aligned} &A_1 = \mathrm{affine}(X, (W_1, \mathbf{b}_1)) \\ &Z = \mathrm{relu}(A_1) \\ &A_2 = \mathrm{affine}(Z, (W_2, \mathbf{b}_2)) \end{aligned}}$$
$$L = \mathrm{loss}(A_2, T, (W_1, W_2))$$

$$L.\,\mathrm{back}(A_2, T, (W_1, W_2))$$

$$\boxed{\begin{aligned} &A_2.\,\mathrm{back}(Z, (W_2, \mathbf{b}_2)) \\ \\ &Z.\,\mathrm{back}(A_1) \\ &A_1.\,\mathrm{back}(X, (W_1, \mathbf{b}_1)) \end{aligned}}$$

def $\mathrm{model}(X, (U_1, U_2))$:

$$\boxed{\begin{aligned} &A_1 = \mathrm{affine}(X, U_1) \\ &Z = \mathrm{relu}(A) \\ &A_2 = \mathrm{affine}(Z, U_2) \end{aligned}}$$

def $\mathrm{back}(X, (U_1, U_2))$:

$$\boxed{\begin{aligned} &A_2.\,\mathrm{back}(Z, U_2) \\ &Z.\,\mathrm{back}(A) \\ &A_1.\,\mathrm{back}(X, U_1) \end{aligned}}$$

return $\mathit{block}(A_2, \mathrm{back})$

automatic differentiation

$\boxed{A_2 = \mathrm{model}(X, ((W_1, \mathbf{b}_1), (W_2, \mathbf{b}_2)))}$

$L = \mathrm{loss}(A_2, T, (W_1, W_2))$

$L.\,\mathrm{back}(A_2, T, (W_1, W_2))$

$\boxed{A_2.\,\mathrm{back}(X, ((W_1, \mathbf{b}_1), (W_2, \mathbf{b}_2)))}$

def $\mathrm{model}(X, (U_1, U_2))$:
    $A_1 = \mathrm{affine}(X, U_1)$
    $Z = \mathrm{relu}(A)$
    $A_2 = \mathrm{affine}(Z, U_2)$
    def $\mathrm{back}(X, (U_1, U_2))$:
        $A_2.\,\mathrm{back}(Z, U_2)$
        $Z.\,\mathrm{back}(A)$
        $A_1.\,\mathrm{back}(X, U_1)$
    return $\mathit{block}(A_2, \mathrm{back})$

automatic differentiation

$A_2 = \mathrm{model}(X, ((W_1, \mathbf{b}_1), (W_2, \mathbf{b}_2)))$

$L = \mathrm{loss}(A_2, T, (W_1, W_2))$

def $\mathrm{model}(X, (U_1, U_2))$:
  $A = \mathrm{affine}(\mathrm{relu}(\mathrm{affine}(X, U_1)), U_2)$
  return $\mathit{block}(A)$

$L.\,\mathrm{back}(A_2, T, (W_1, W_2))$

def $\mathrm{model}(X, (U_1, U_2))$:
  $A_1 = \mathrm{affine}(X, U_1)$
  $Z = \mathrm{relu}(A)$
  $A_2 = \mathrm{affine}(Z, U_2)$
  ~~**def** $\mathrm{back}(X, (U_1, U_2))$:~~
    ~~$A_2.\,\mathrm{back}(Z, U_2)$~~
    ~~$Z.\,\mathrm{back}(A)$~~
    ~~$A_1.\,\mathrm{back}(X, U_1)$~~
  return $\mathit{block}(A_2, \mathrm{back})$

$A_2.\,\mathrm{back}(X, ((W_1, \mathbf{b}_1), (W_2, \mathbf{b}_2)))$

convolution

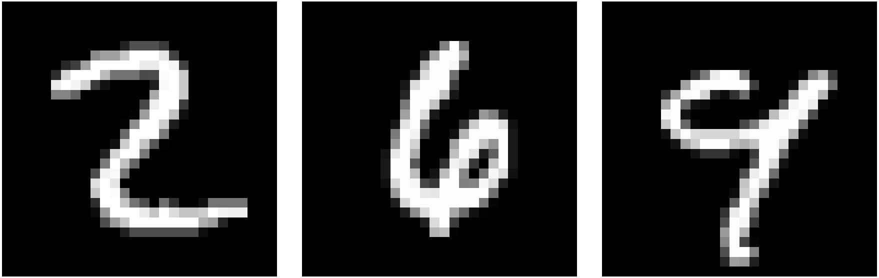
MNIST digits dataset

- 10 classes, 60k training images, 10k test images, 28×28 images

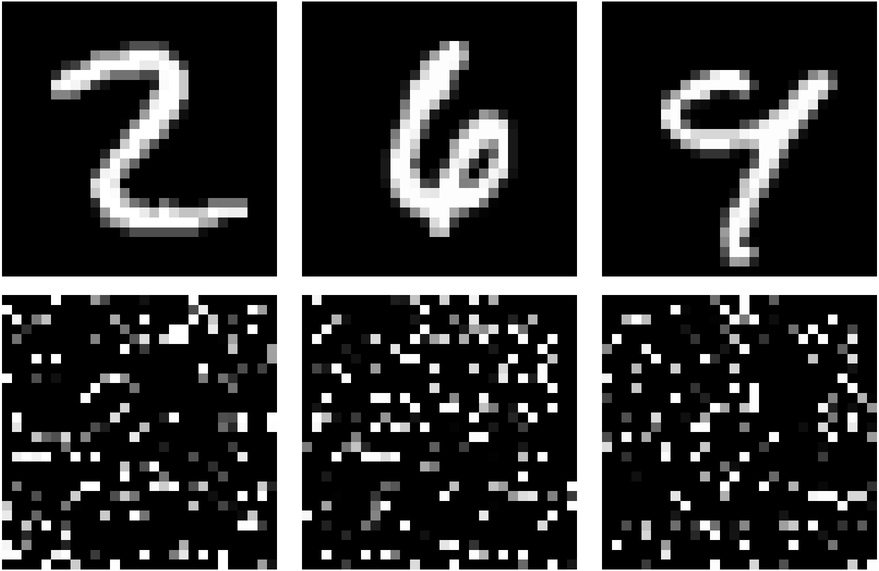
fully connected layers

- a two-layer network with fully connected layers can easily learn to classify MNIST digits (less that 3% error), but learns more than actually required

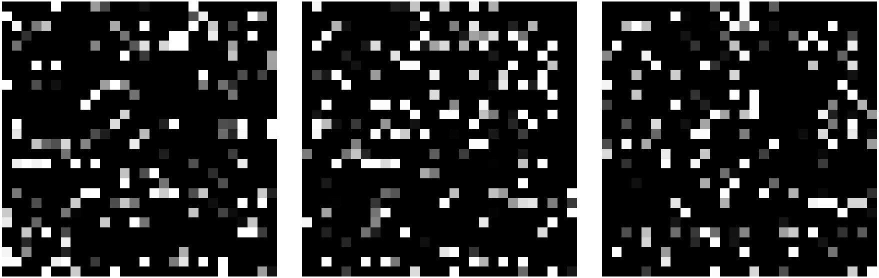
shuffling the dimensions



shuffling the dimensions



shuffling the dimensions



convolution

- convolution results in sparser connections between units, local receptive fields, translation equivariance, shared weights and less parameters to learn
- a convolutional network performs better (less than 1% error), but not on shuffled digits

convolution

- convolution results in sparser connections between units, local receptive fields, translation equivariance, shared weights and less parameters to learn
- a convolutional network performs better (less than 1% error), but not on shuffled digits

LTI systems and convolution

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
- translation (or shift, or delay) $t_k(x)[n] = x[n-k]$, $k \in \mathbb{Z}$
- unit impulse $\delta[n] = \mathbb{1}[n=0]$, so that $x[n] = \sum_k x[k]\delta[n-k]$
- linear system (or filter)

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

- time-invariant (or translation equivariant) system

$$f(t_k(x)) = t_k(f(x))$$

- if f is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$f(x)[n] = f\left(\sum_k x[k] t_k(\delta)\right)[n] = \sum_k x[k] t_k(f(\delta))[n]$$
$$= \sum_k x[k] h[n-k]$$

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- if f is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$\begin{aligned} f(x)[n] &= f\left(\boxed{\sum_k x[k]t_k(\delta)}\right)[n] = \sum_k x[k]t_k(f(\delta))[n] \\ &= \sum_k x[k]h[n-k] \end{aligned}$$

LTI systems and convolution

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
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$$f(t_k(x)) = t_k(f(x))$$

- if f is LTI with impulse response $h = f(\delta)$, then $f(x) = x * h$:

$$\begin{aligned} f(x)[n] &= \boxed{f}\left(\sum_k x[k] t_k(\delta)\right)[n] = \sum_k x[k] t_k\boxed{(f(}\delta))[n] \\ &= \sum_k x[k] h[n-k] \end{aligned}$$

LTI systems and convolution

- discrete-time signal: $x[n]$, $n \in \mathbb{Z}$
- translation (or shift, or delay) $t_k(x)[n] = x[n-k]$, $k \in \mathbb{Z}$
- unit impulse $\delta[n] = \mathbb{1}[n=0]$, so that $x[n] = \sum_k x[k]\delta[n-k]$
- linear system (or filter)

$$f\left(\sum_i a_i x_i\right) = \sum_i a_i f(x_i)$$

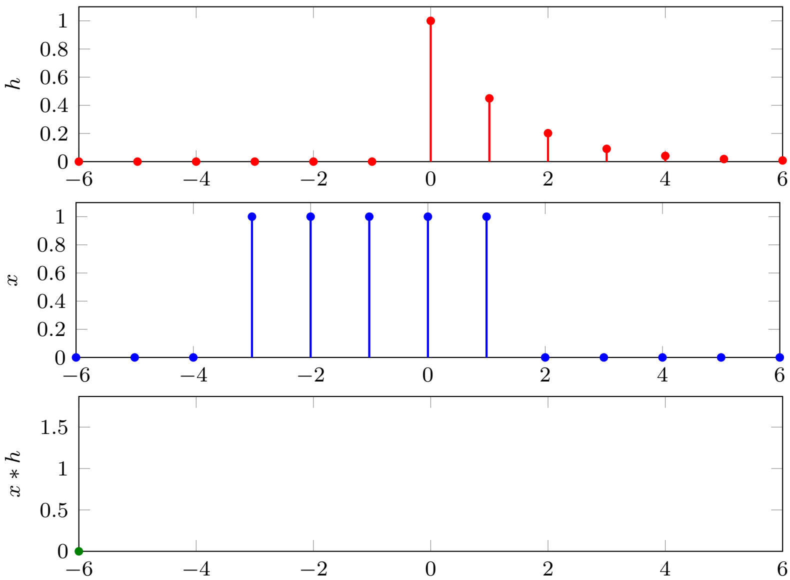
- time-invariant (or translation equivariant) system

$$f(t_k(x)) = t_k(f(x))$$

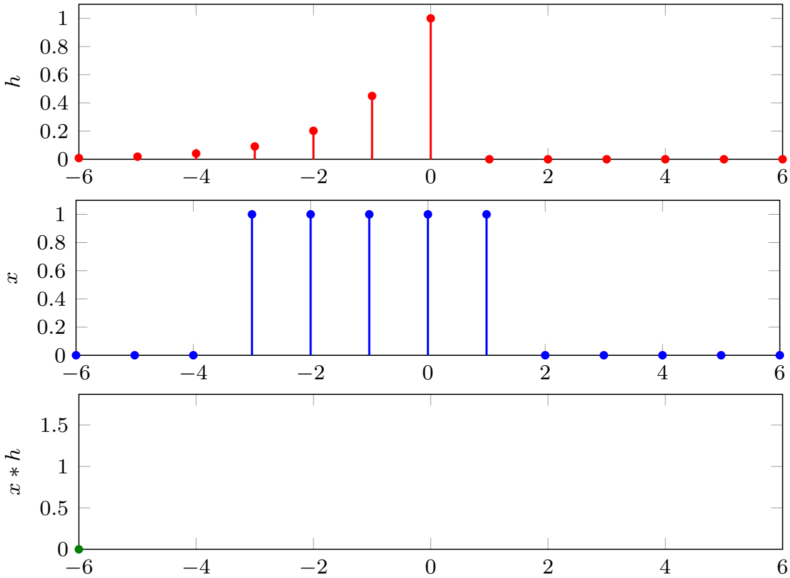
- if f is LTI with impulse response $h = f(\delta)$, then $f(x) = \boxed{x * h}$:

$$f(x)[n] = f\left(\sum_k x[k] t_k(\delta)\right)[n] = \sum_k x[k] t_k(f(\delta))[n]$$
$$= \boxed{\sum_k x[k]h[n-k]}$$

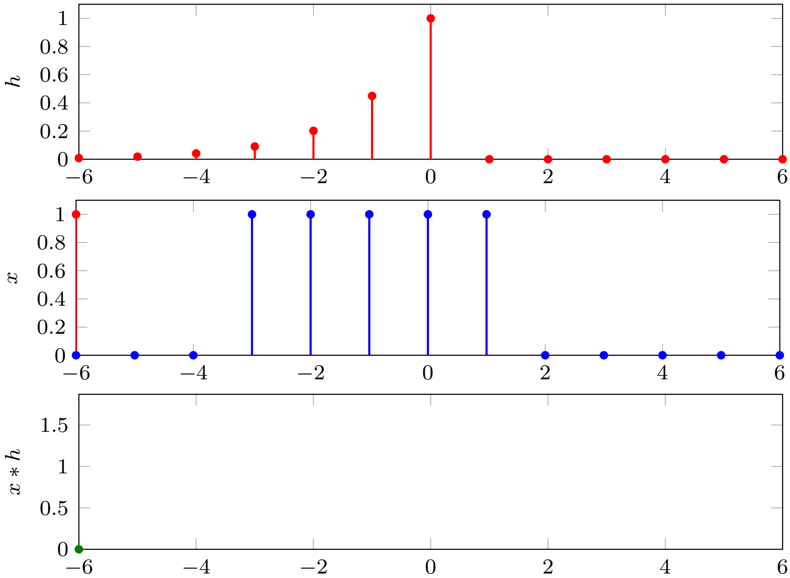
convolution



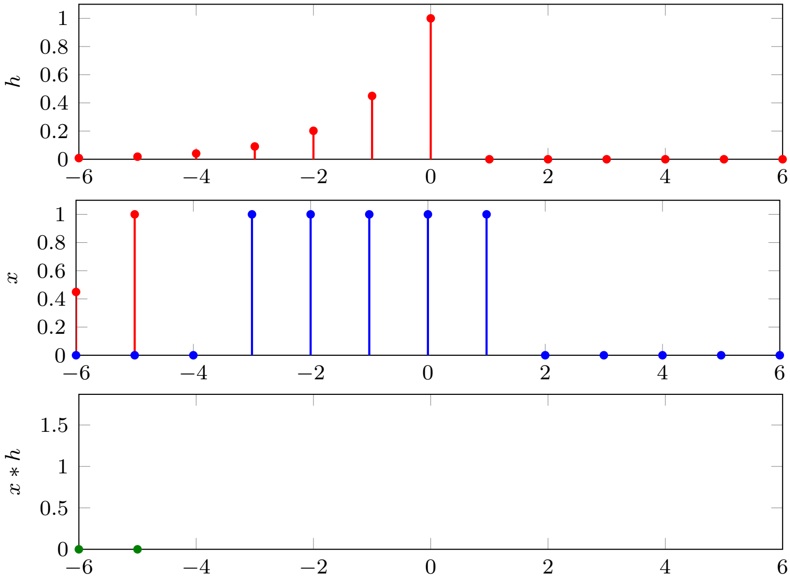
convolution



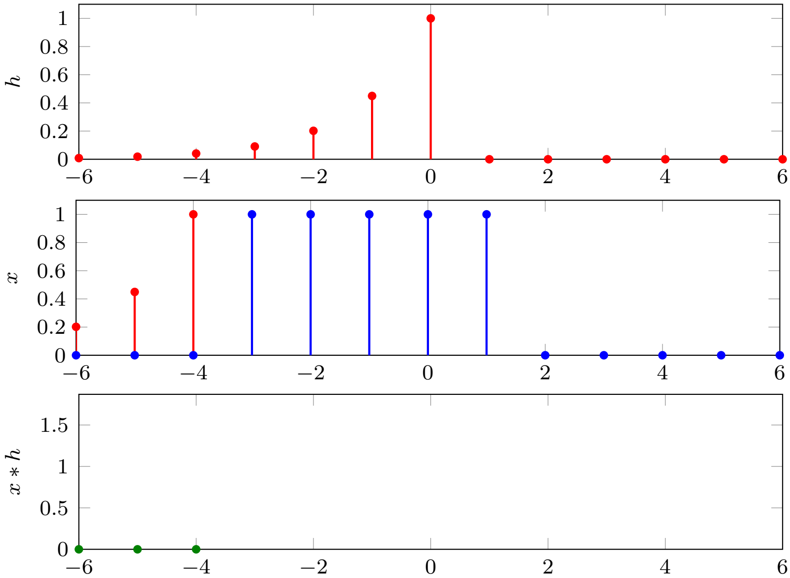
convolution



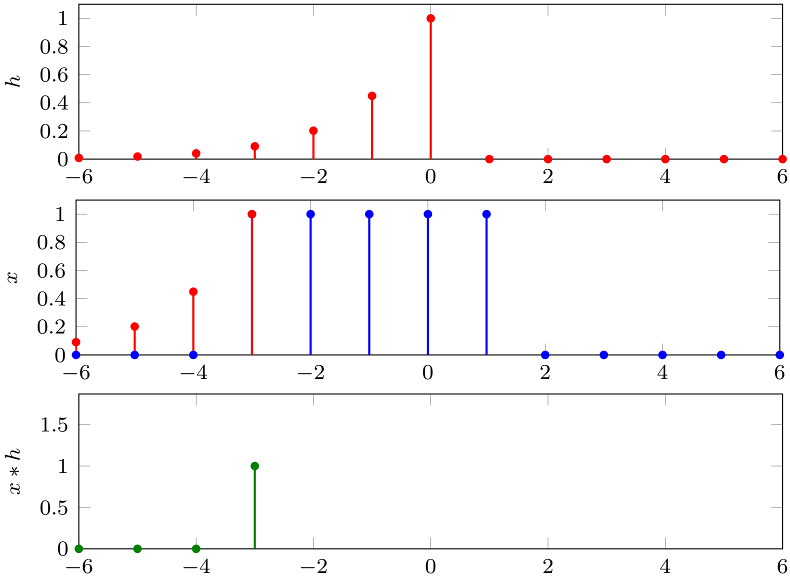
convolution



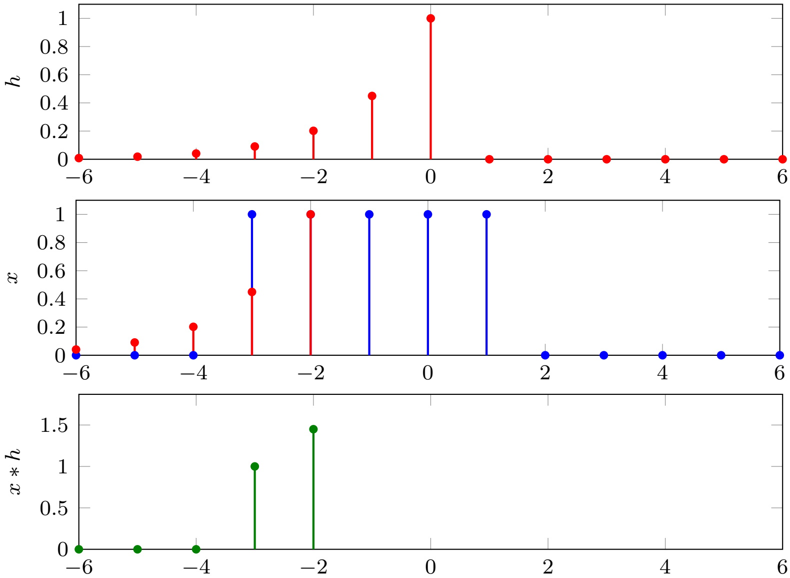
convolution



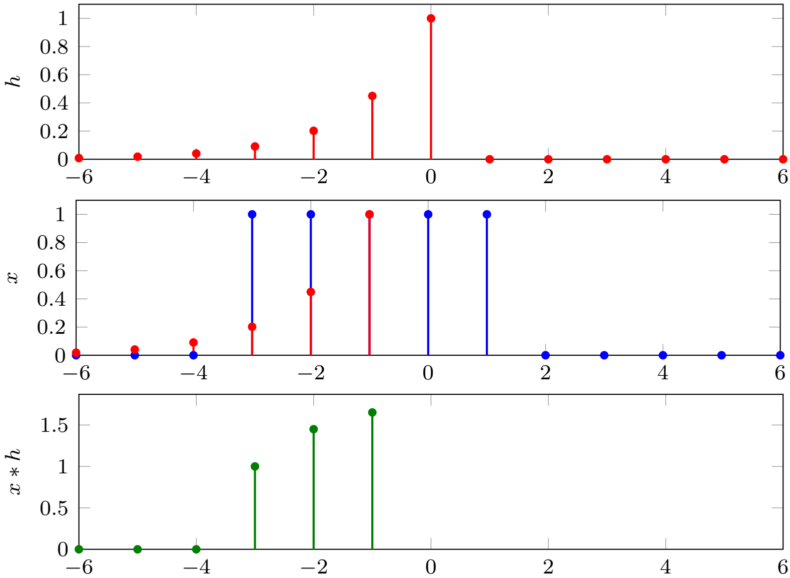
convolution



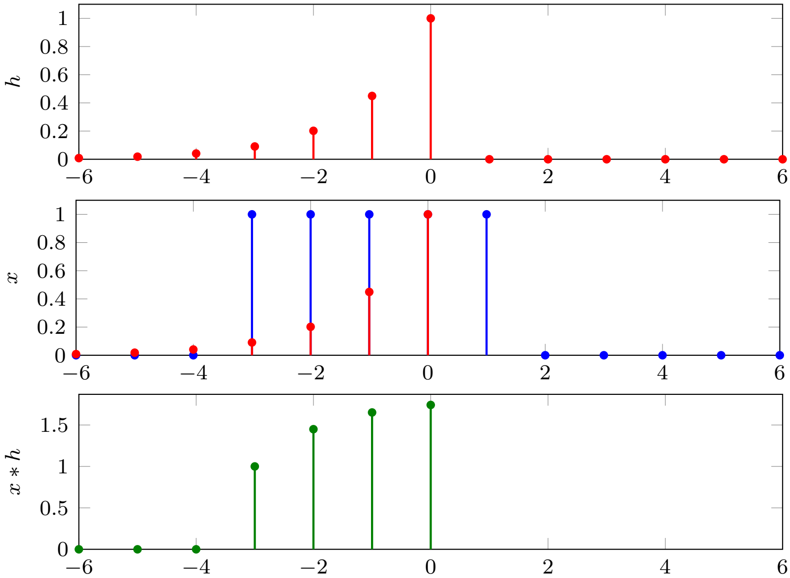
convolution



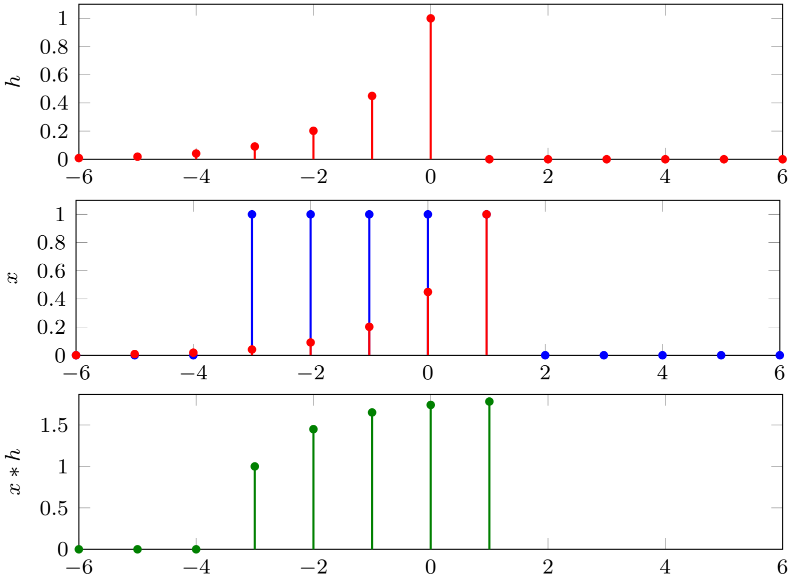
convolution



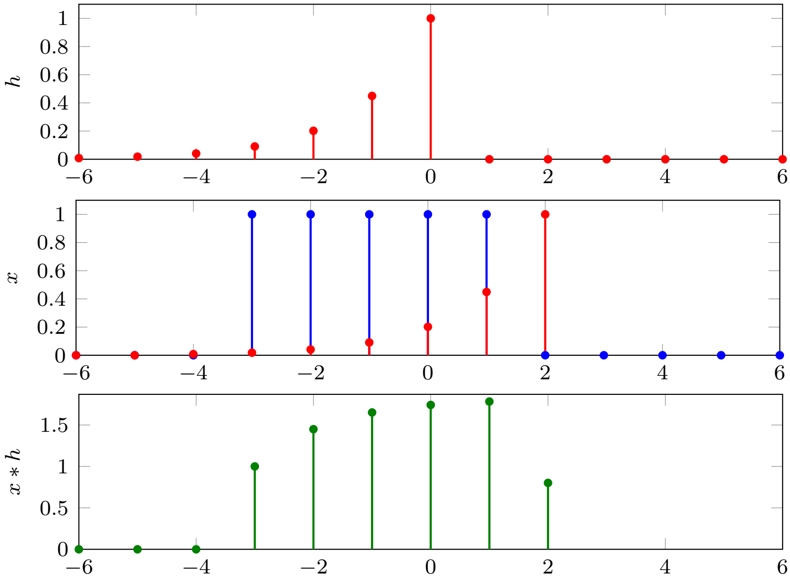
convolution



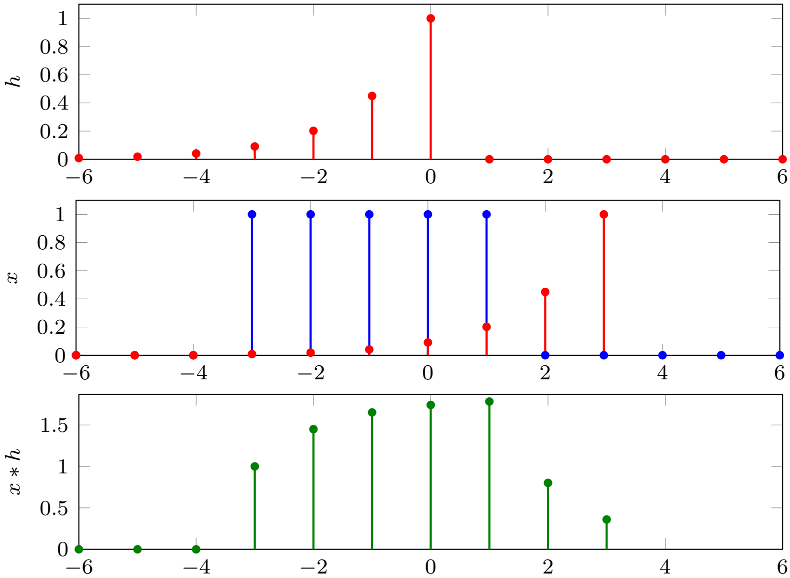
convolution



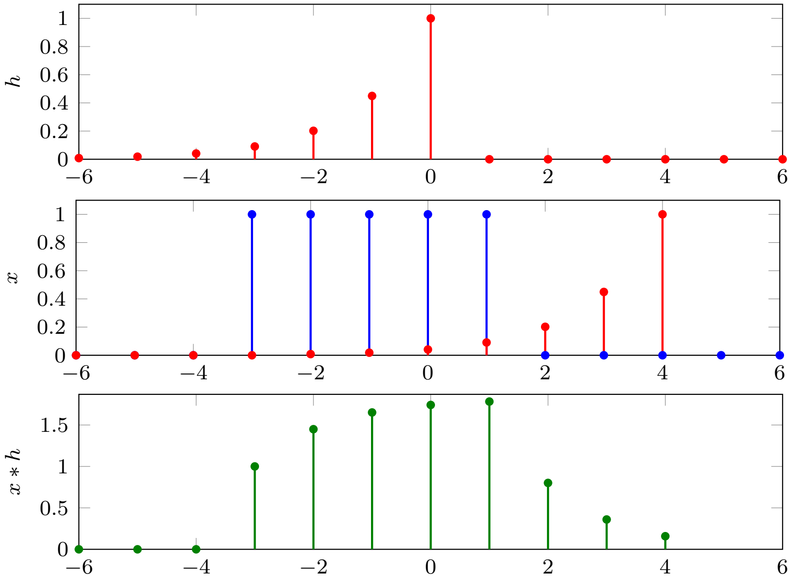
convolution



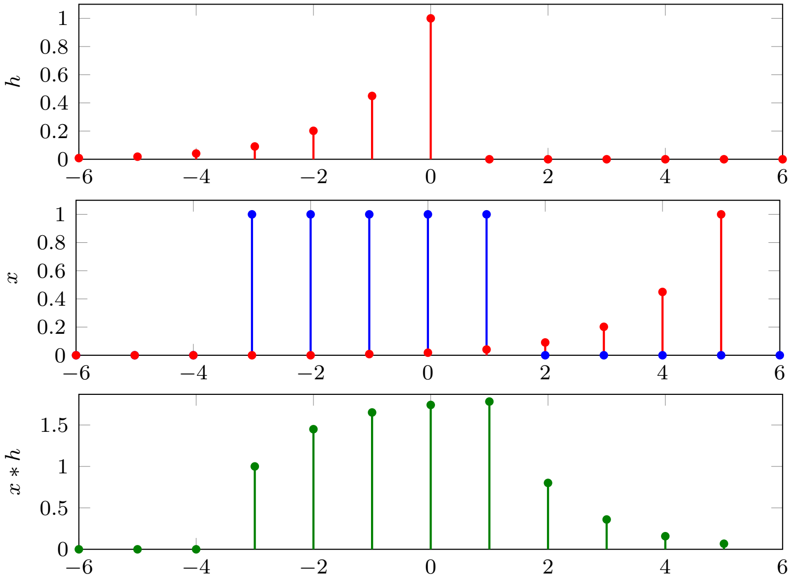
convolution



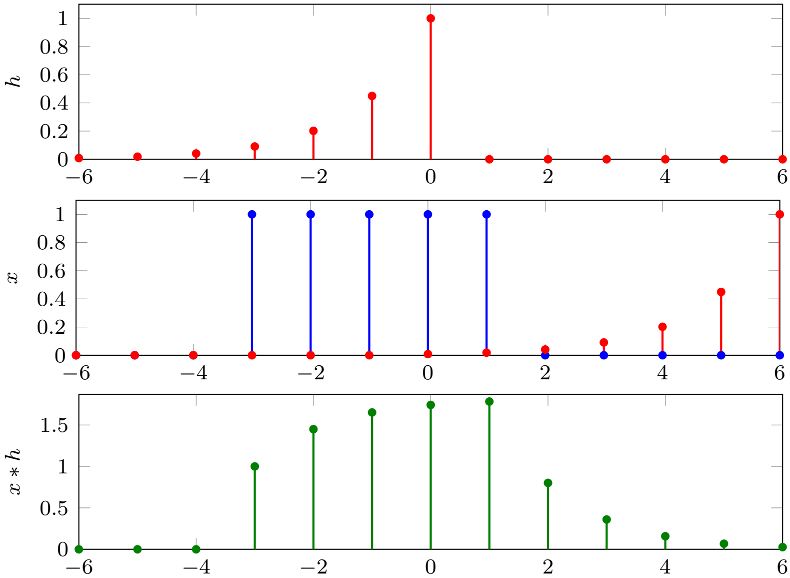
convolution



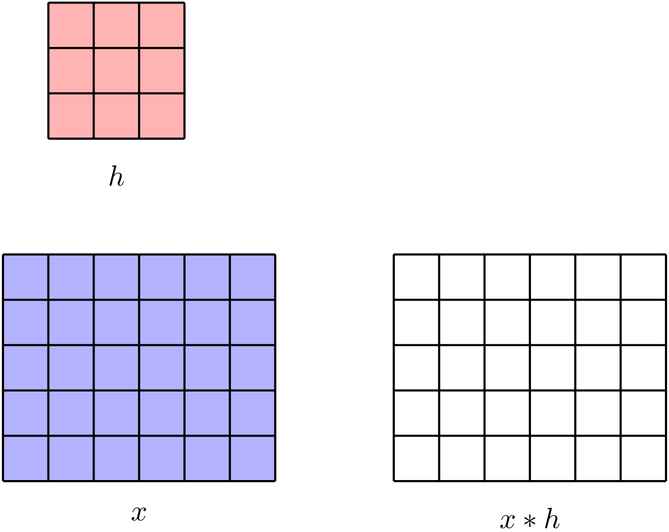
convolution



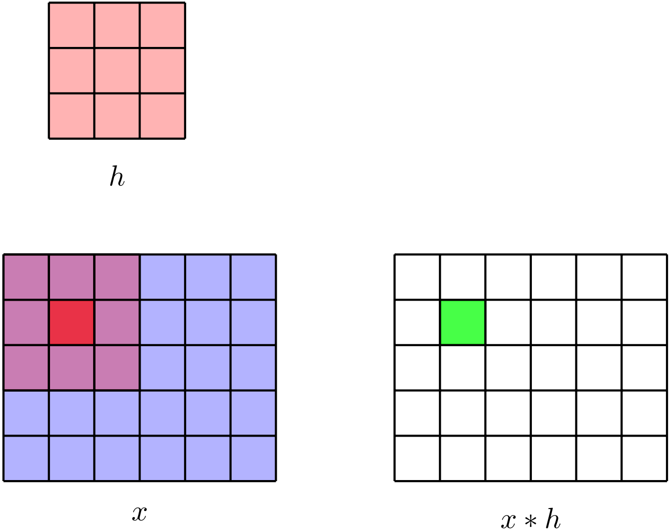
convolution



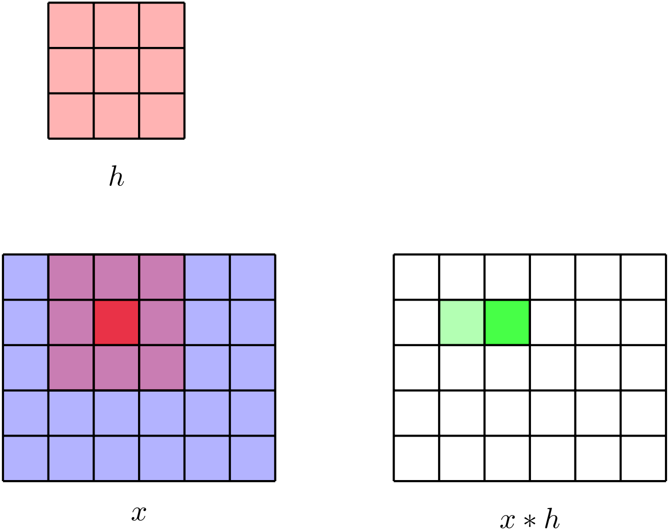
2d convolution



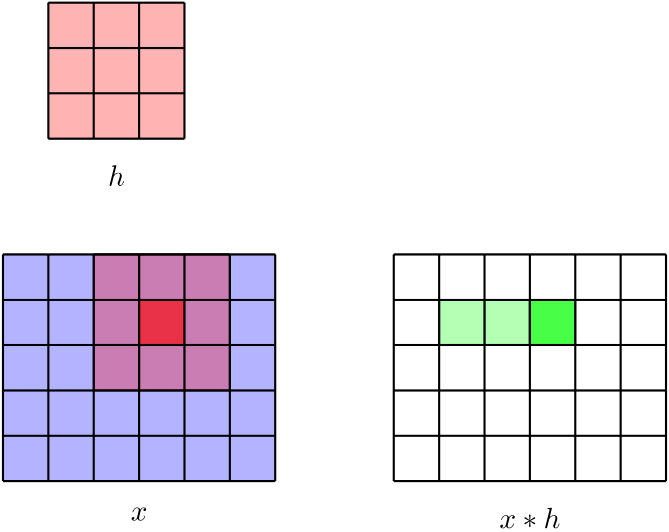
2d convolution



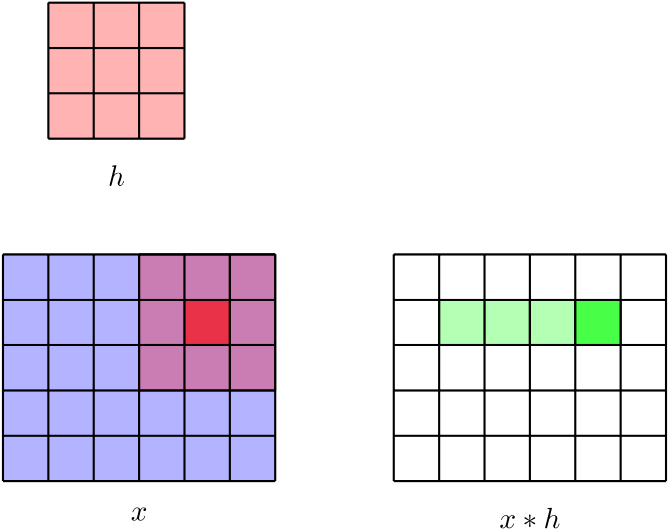
2d convolution



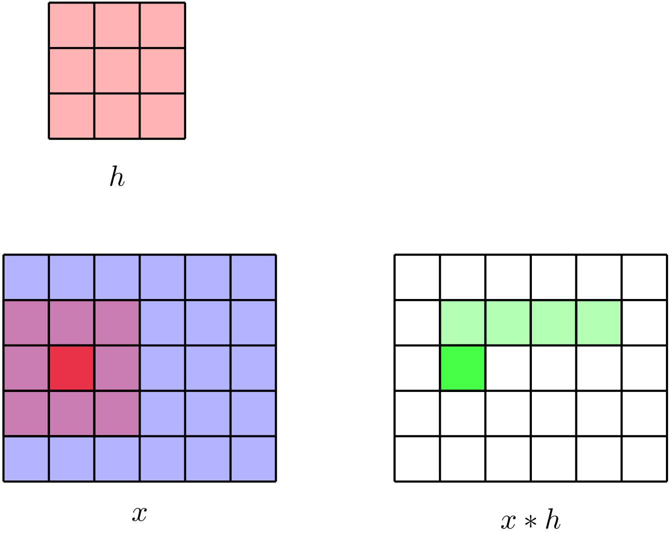
2d convolution



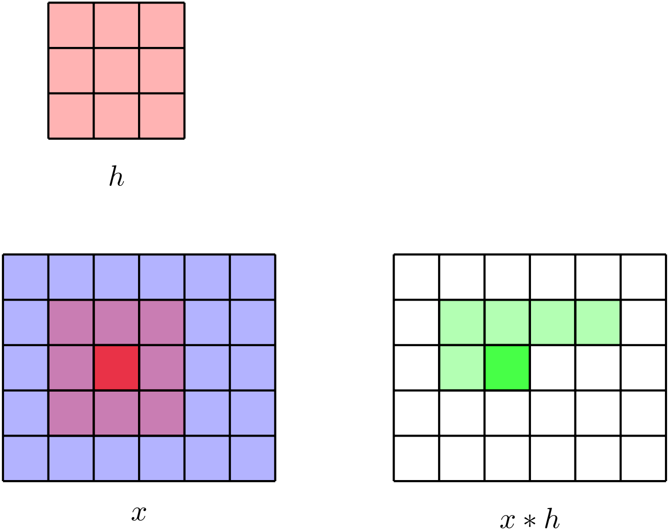
2d convolution



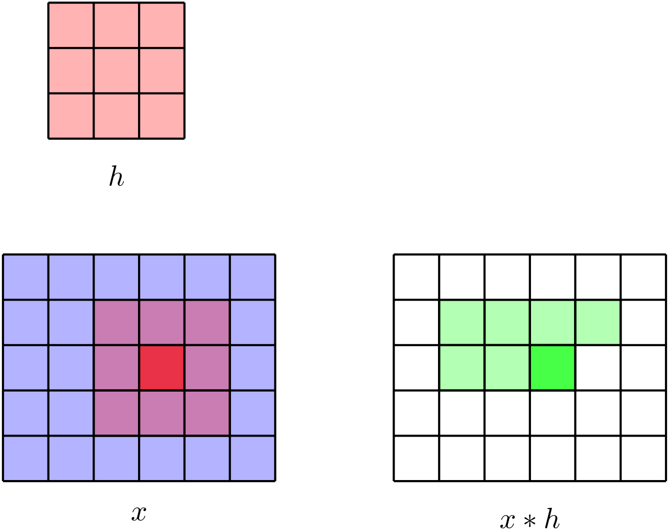
2d convolution



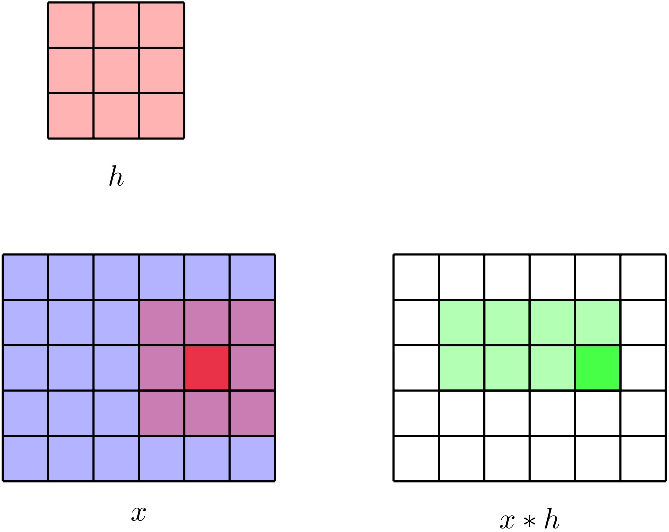
2d convolution



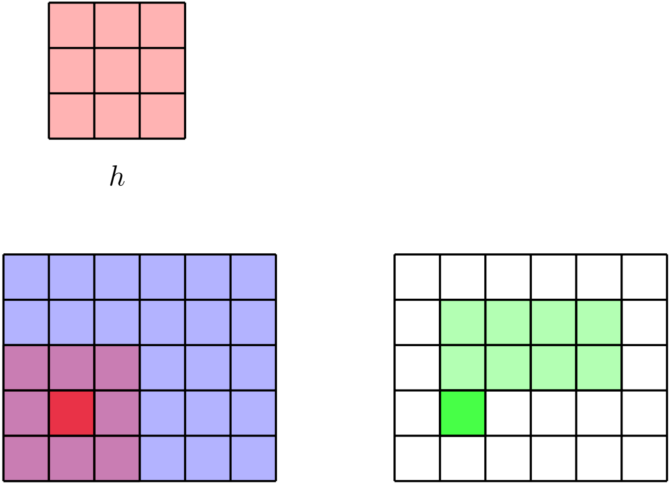
2d convolution



2d convolution



2d convolution

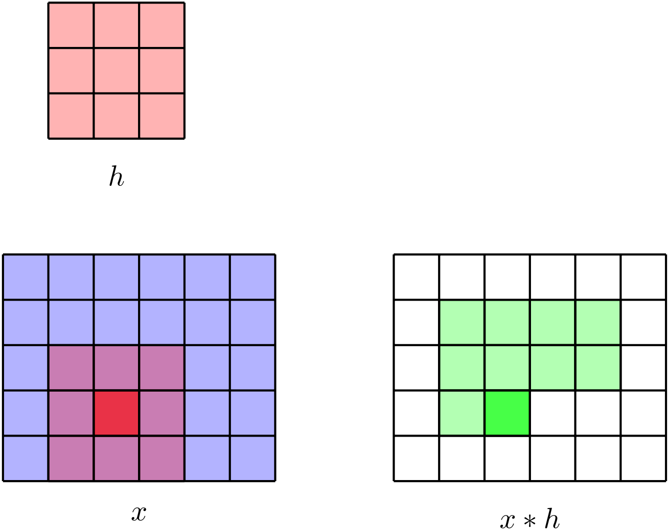


h

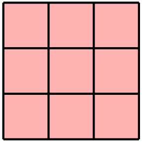
x

$x * h$

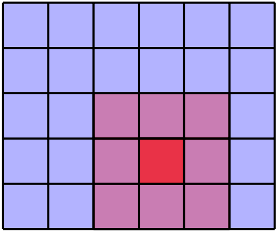
2d convolution



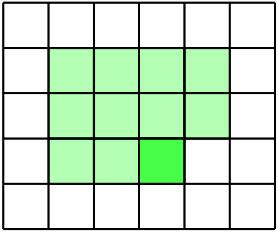
2d convolution



h

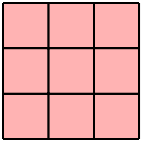


x

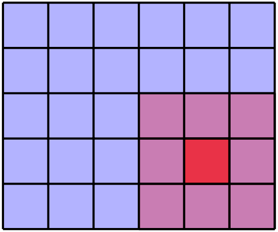


$x * h$

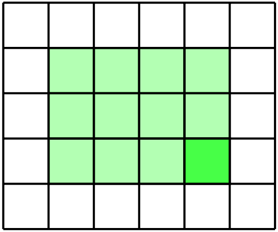
2d convolution



h

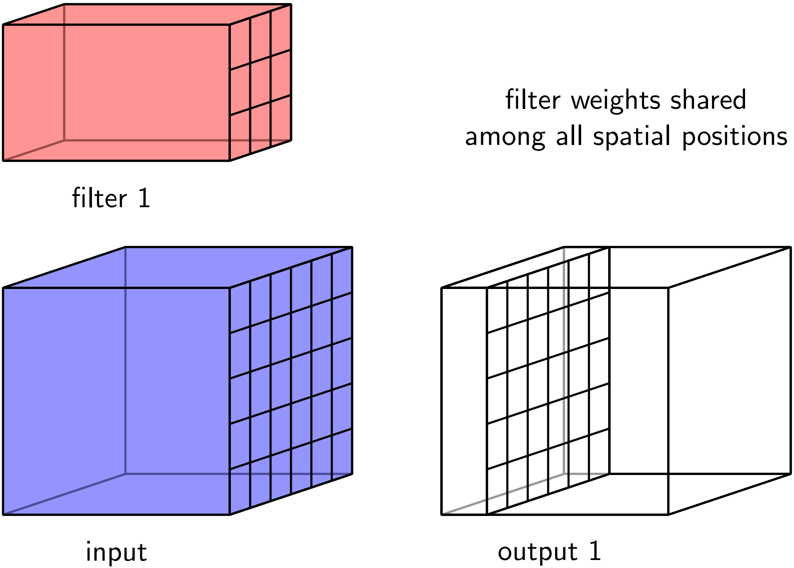


x



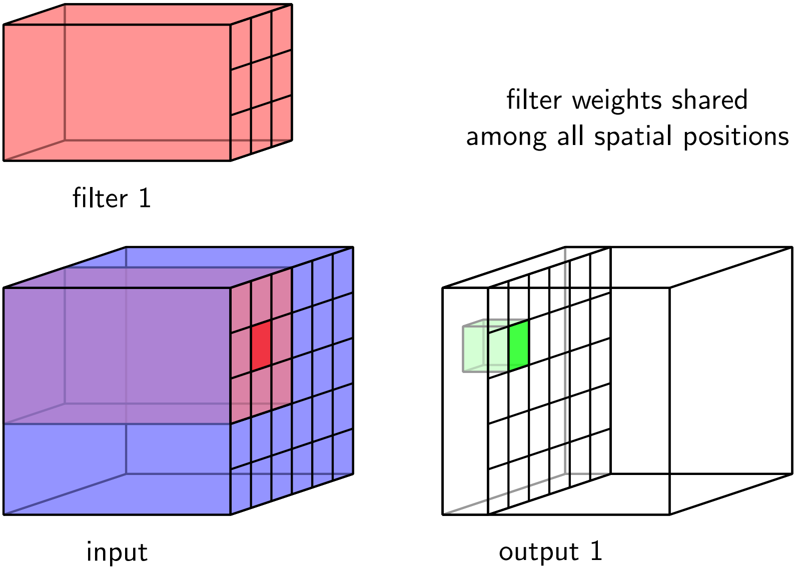
$x * h$

convolution in feature maps



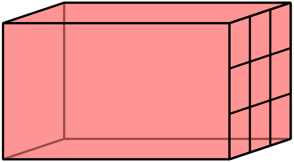
filter weights shared
among all spatial positions

convolution in feature maps



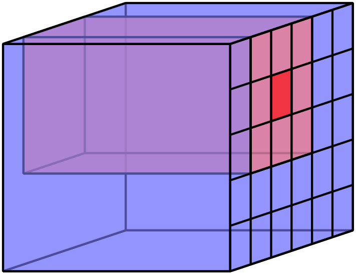
filter weights shared
among all spatial positions

convolution in feature maps

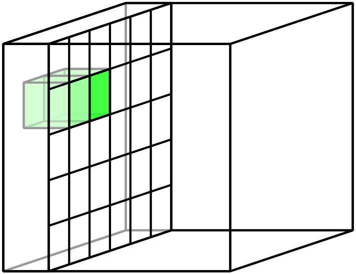


filter 1

filter weights shared
among all spatial positions

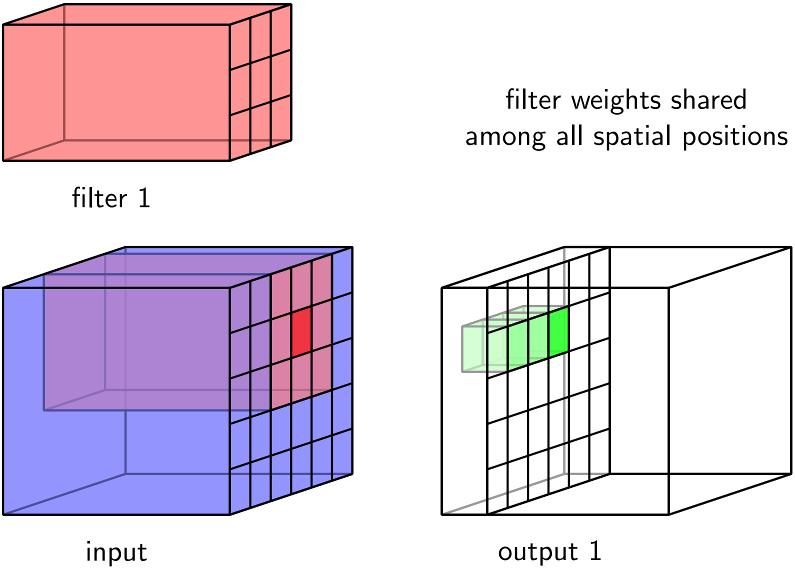


input



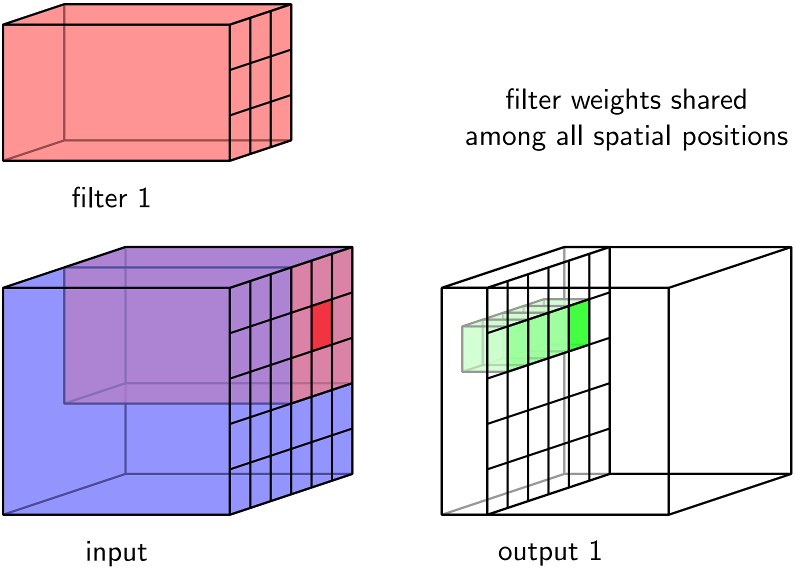
output 1

convolution in feature maps

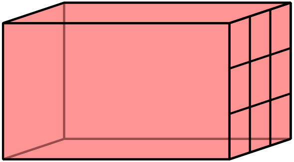


filter weights shared
among all spatial positions

convolution in feature maps

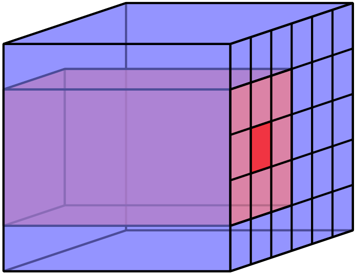


convolution in feature maps

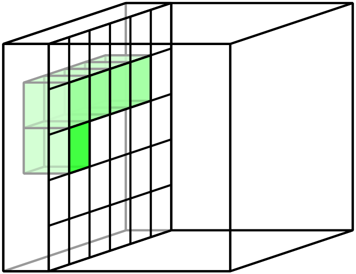


filter 1

filter weights shared
among all spatial positions

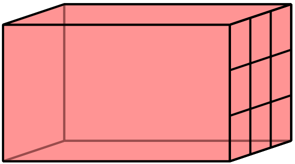


input



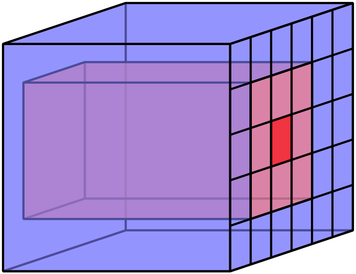
output 1

convolution in feature maps

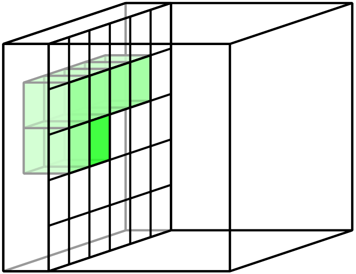


filter 1

filter weights shared
among all spatial positions

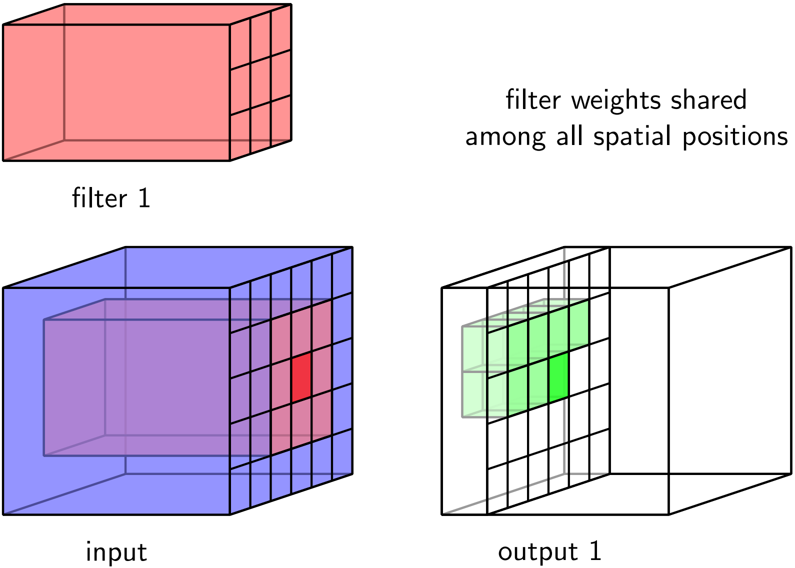


input

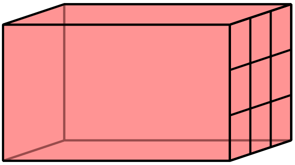


output 1

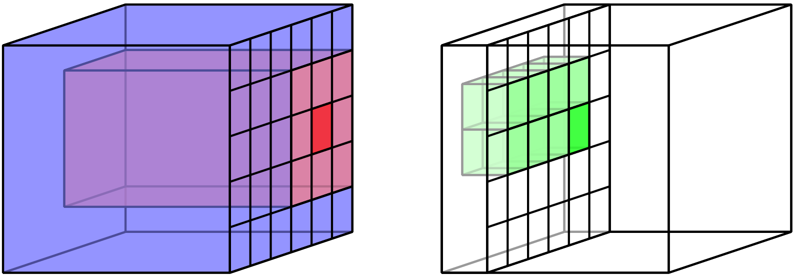
convolution in feature maps



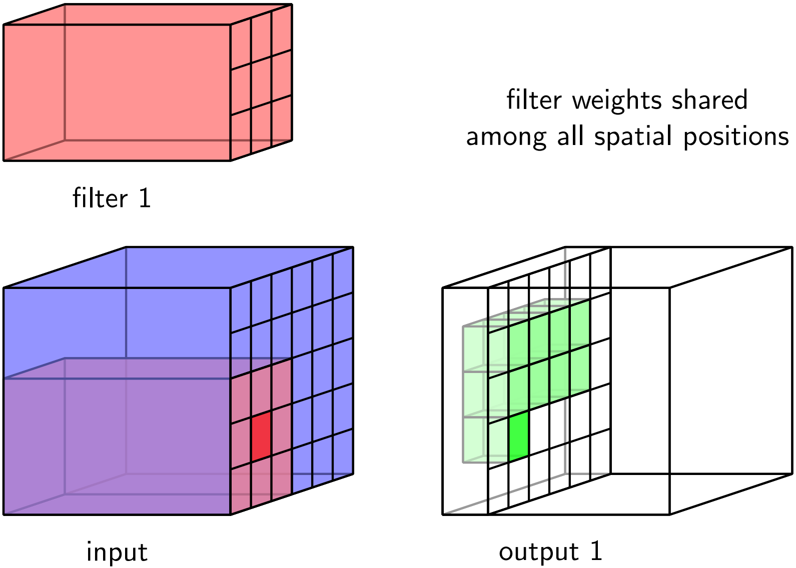
convolution in feature maps



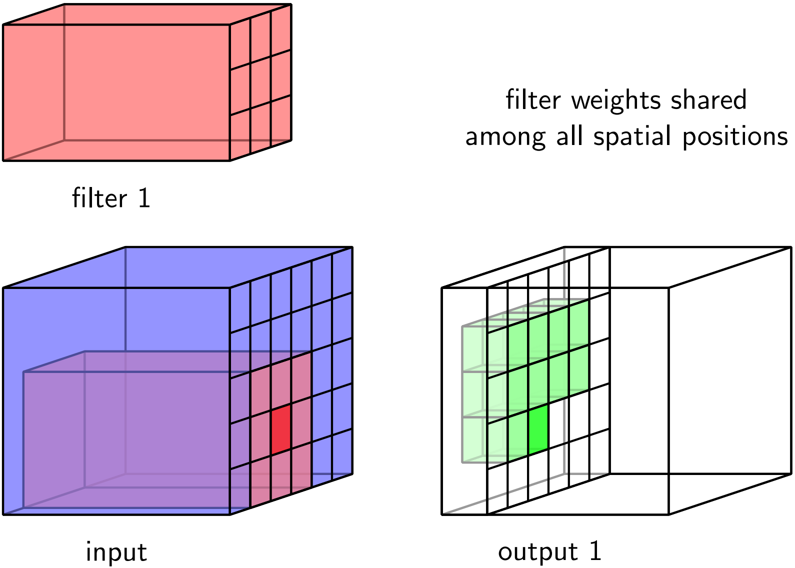
filter weights shared
among all spatial positions



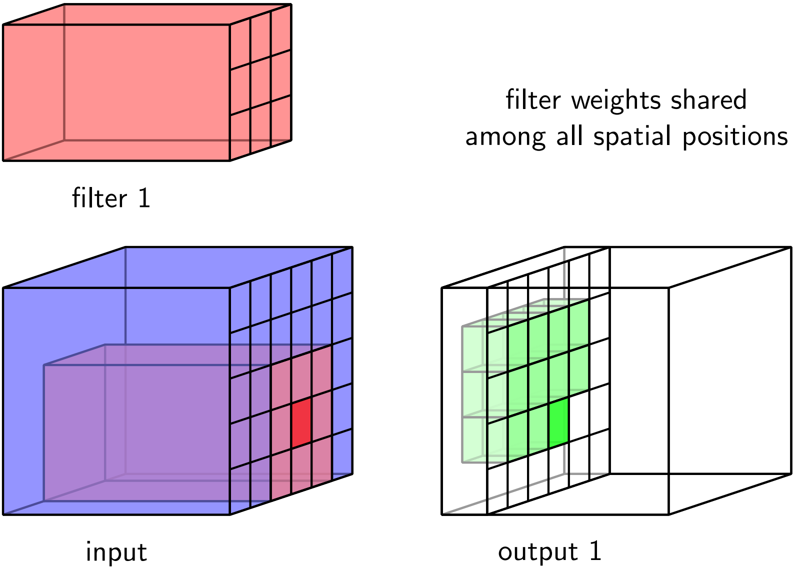
convolution in feature maps



convolution in feature maps

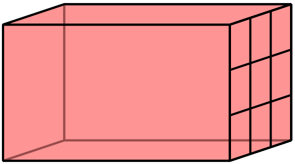


convolution in feature maps



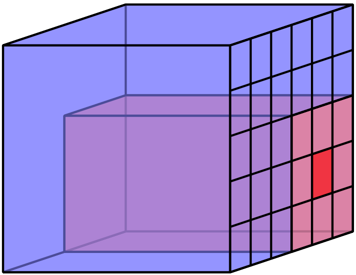
filter weights shared
among all spatial positions

convolution in feature maps

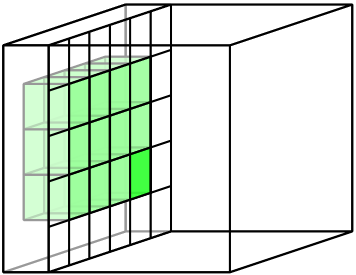


filter 1

filter weights shared
among all spatial positions

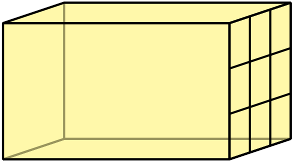


input



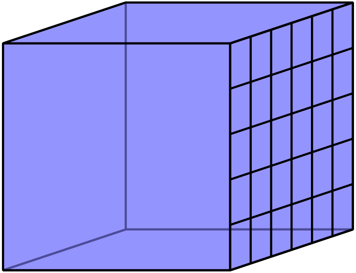
output 1

convolution in feature maps

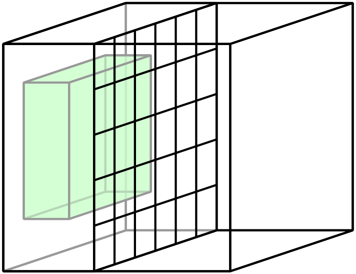


filter 2

new filter, but still shared
among all spatial positions

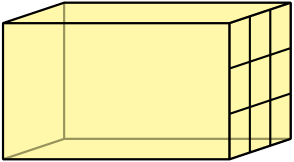


input



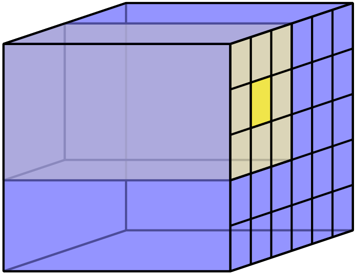
output 2

convolution in feature maps

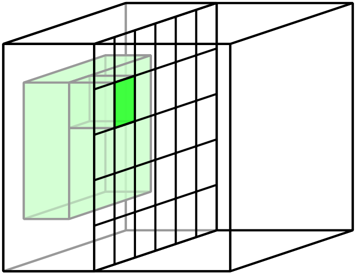


filter 2

new filter, but still shared among all spatial positions

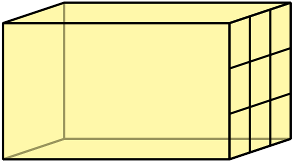


input



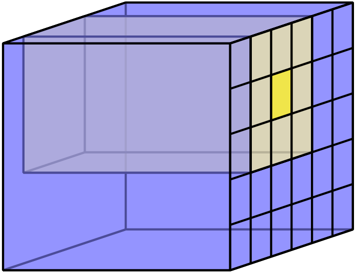
output 2

convolution in feature maps

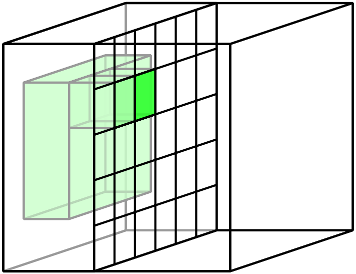


filter 2

new filter, but still shared
among all spatial positions

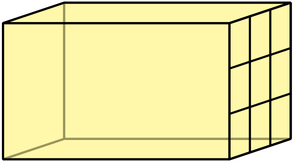


input



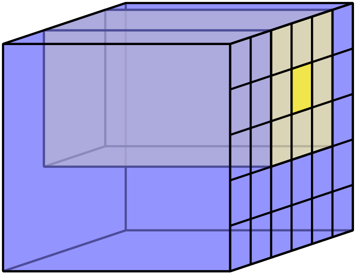
output 2

convolution in feature maps

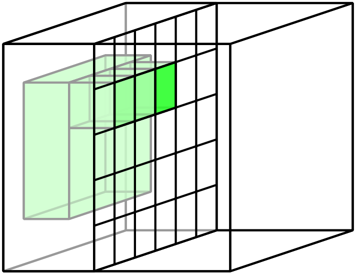


filter 2

new filter, but still shared
among all spatial positions

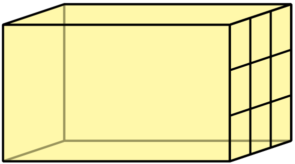


input



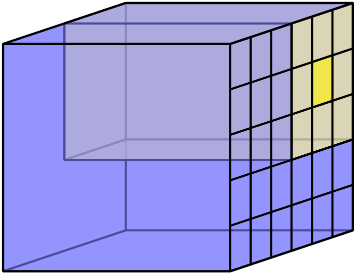
output 2

convolution in feature maps

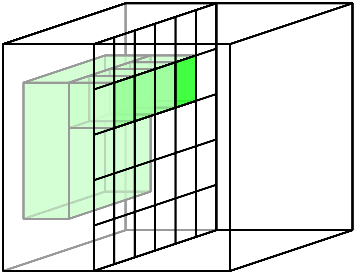


filter 2

new filter, but still shared
among all spatial positions

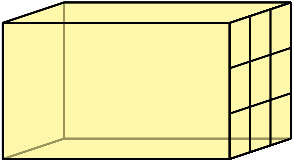


input



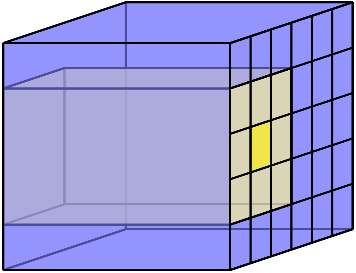
output 2

convolution in feature maps



filter 2

new filter, but still shared
among all spatial positions



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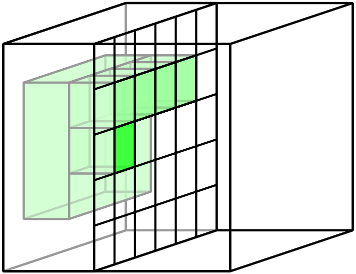

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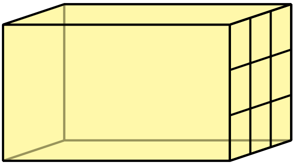

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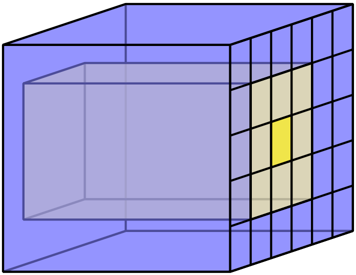

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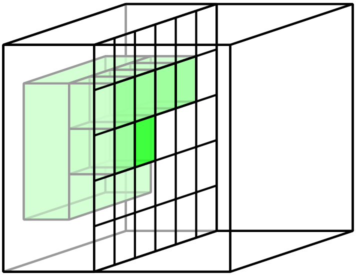

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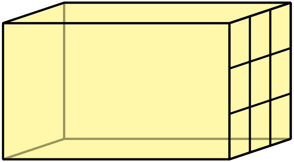

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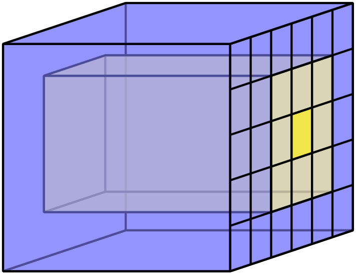

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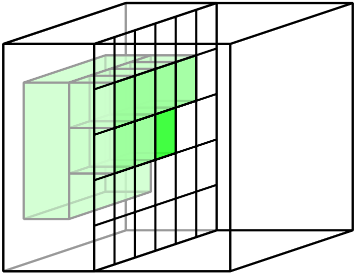

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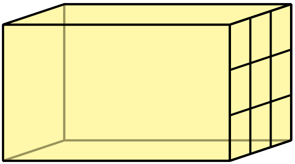

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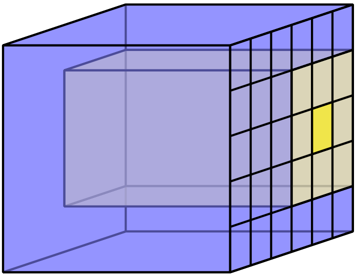

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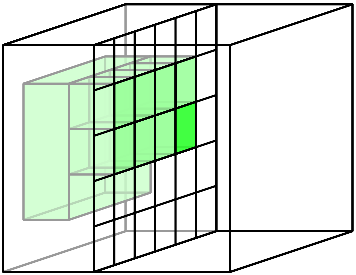

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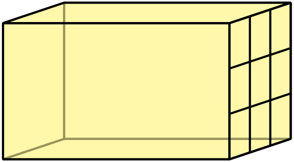

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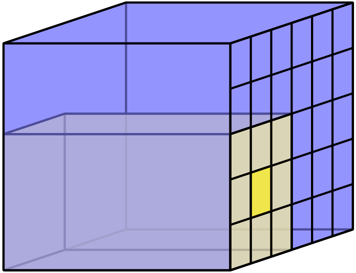

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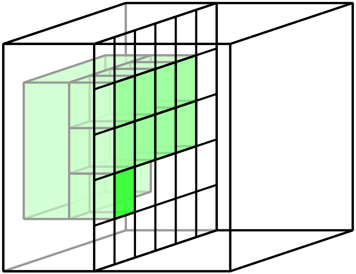

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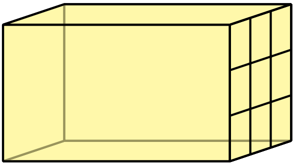

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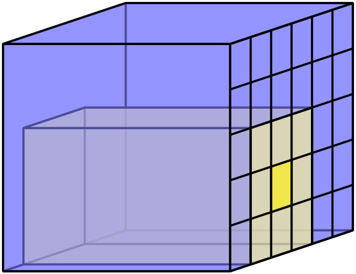

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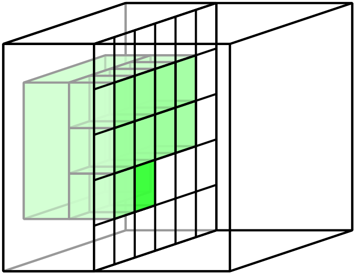

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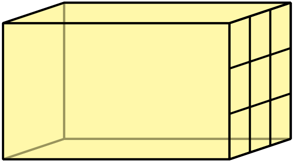

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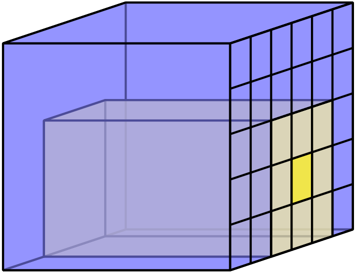
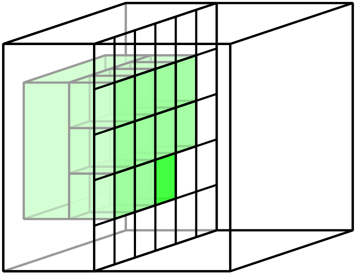


input



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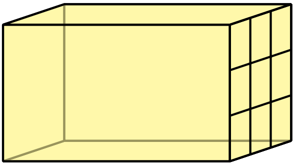

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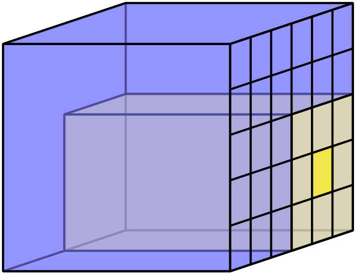

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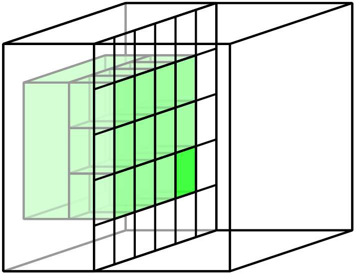
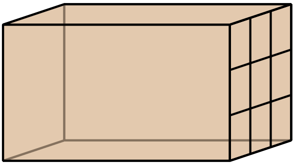


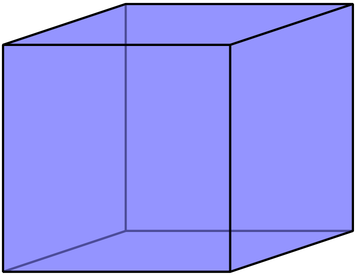
output 2

convolution in feature maps

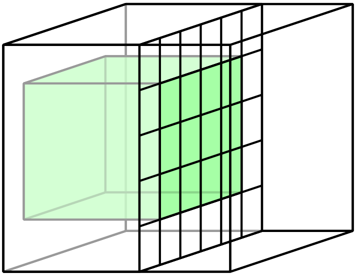


filter 3

different filter for each
output dimension

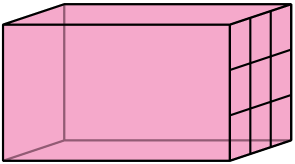


input



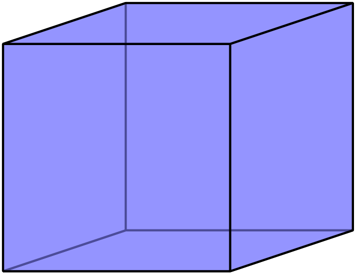
output 3

convolution in feature maps

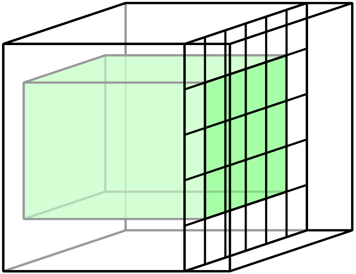


filter 4

different filter for each output dimension



input

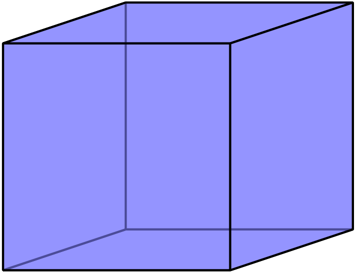


output 4

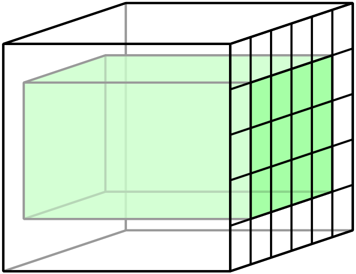
convolution in feature maps

filter 5

different filter for each output dimension



input

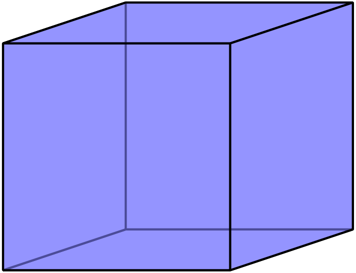


output 5

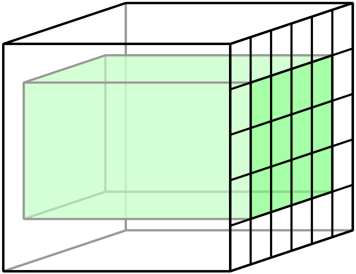
convolution in feature maps

filter 5

1×1 filter is matrix multiplication



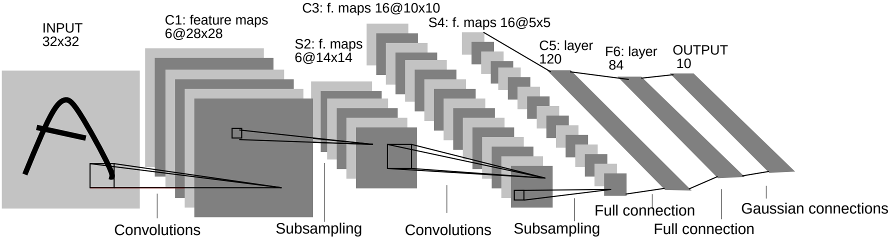
input



output 5

LeNet-5

[LeCun et al. 1998]



- sub-sampling gradually introduces translation, scale and distortion invariance
- non-linearity included in sub-sampling layers as feature maps are increasing in dimension

Lecun, Bottou, Bengio, Haffner. IEEE Proc. 1998. Gradient-Based Learning Applied to Document Recognition.

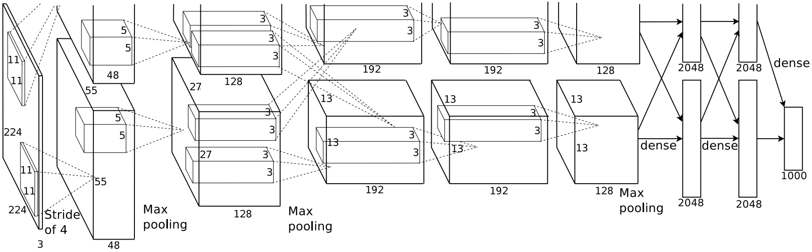
ImageNet

- 22k classes, 15M samples
- ImageNet Large-Scale Visual Recognition Challenge (ILSVRC): 1000 classes, 1.2M training images, 50k validation images, 150k test images

Russakovsky, Deng, Su, Krause, *et al.* 2014. Imagenet Large Scale Visual Recognition Challenge.

AlexNet

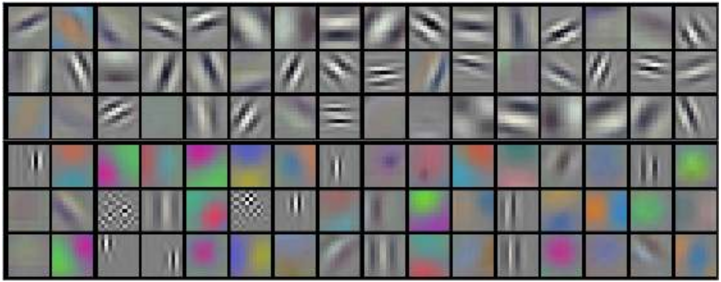
[Krizhevsky et al. 2012]



- implementation on two GPUs; connectivity between the two subnetworks is limited
- ReLU, data augmentation, local response normalization, dropout
- outperformed all previous models on ILSVRC by 10%

learned layer 1 kernels

[Krizhevsky et al. 2012]



- 96 kernels of size $11 \times 11 \times 3$
- top: 48 GPU 1 kernels; bottom: 48 GPU 2 kernels

Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

visualizing intermediate layers

[Zeiler and Fergus 2014]

- reconstructed patterns from top 9 activations of selected features of layer 4 and corresponding image patches

Zeiler, Fergus. ECCV 2014. Visualizing and Understanding Convolutional Networks.

challenges and applications

challenges

- optimizing
- initializing
- regularizing
- enabling deeper networks
- learning activation functions
- learning the architecture
- designing task-specific architectures and loss functions
- transferring to new domains and tasks
- learning without supervision

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first-order optimization

- loss function

$$L = F(\mathbf{X}, \mathbf{T}; \boldsymbol{\theta}) = \sum_{n \in [N]} f(\mathbf{x}_i, \mathbf{t}_i; \boldsymbol{\theta}) = \sum_{n \in [N]} f_n(\boldsymbol{\theta})$$

- (batch) gradient descent

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \epsilon \frac{1}{N} \sum_{n \in [N]} \frac{\partial f_n}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta}^t)$$

- stochastic (mini-batch) gradient descent

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t - \epsilon \frac{1}{|B^t|} \sum_{n \in B^t} \frac{\partial f_n}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta}^t)$$

makes sense when training set is redundant and each each mini-batch is representative of the entire set

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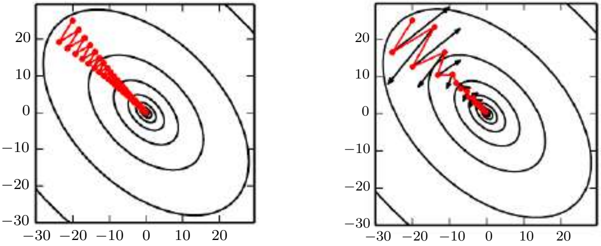
first-order optimization

- momentum: good against noisy gradient and ill-conditioning

$$\mathbf{v}^{t+1} = \mathbf{v}^t - \epsilon \frac{1}{|B^t|} \sum_{n \in B^t} \frac{\partial f_n}{\partial \boldsymbol{\theta}}(\boldsymbol{\theta}^t)$$

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^t + \mathbf{v}^{t+1}$$



- several other methods, but all requiring careful tuning of learning rate

credit: Goodfellow, Bengio, Courville, 2017. Deep learning.

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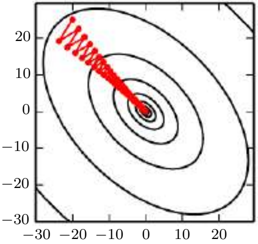

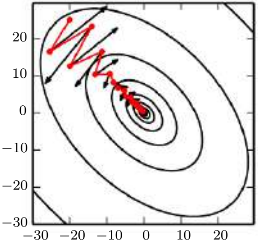

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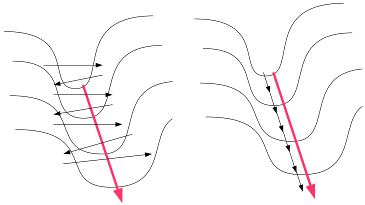
Hessian-free optimization

[Martens ICML 2010]

- Newton's method

$$\boldsymbol{\theta}^{t+1} = \boldsymbol{\theta}^{t} - [\mathbf{H}f(\boldsymbol{\theta}^{t})]^{-1}\nabla f(\boldsymbol{\theta}^{t})$$



- solve linear system

$$[\mathbf{H}f(\boldsymbol{\theta}^{t})]\mathbf{p} = \nabla f(\boldsymbol{\theta}^{t})$$

by conjugate gradient (CG) method, where matrix-vector products of the form $[\mathbf{H}f(\boldsymbol{\theta}^{t})]\mathbf{d}$ are computed by back-propagation

Martens. ICML 2010. Deep Learning via Hessian-Free Optimization.

Hessian-free optimization

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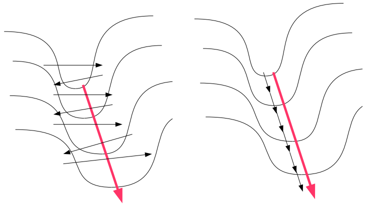

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Martens. ICML 2010. Deep Learning via Hessian-Free Optimization.

batch normalization

[Ioffe and Szegedy 2015]

- samples are element-wise normalized to zero-mean, unit-variance over mini-batch

$$\boldsymbol{\mu}^t \leftarrow \frac{1}{|B^t|} \sum_{i \in B^t} \mathbf{x}_i$$

$$v^t \leftarrow \frac{1}{|B^t|} \sum_{i \in B^t} (\mathbf{x}_i - \boldsymbol{\mu}^t)^2$$

$$\mathbf{y}_i \leftarrow \gamma \frac{\mathbf{x}_i - \boldsymbol{\mu}^t}{\sqrt{v^t + \epsilon}} + \beta$$

- this reduces "internal covariate shift", stabilizing the distribution of each layer's inputs
- it helps with saturating non-linearities and vanishing gradient, allows accelerating learning and reduces the need for regularization

Ioffe and Szegedy. ICML 2015 - Batch Normalization: Accelerating Deep Network Training By Reducing Internal Covariate Shift.

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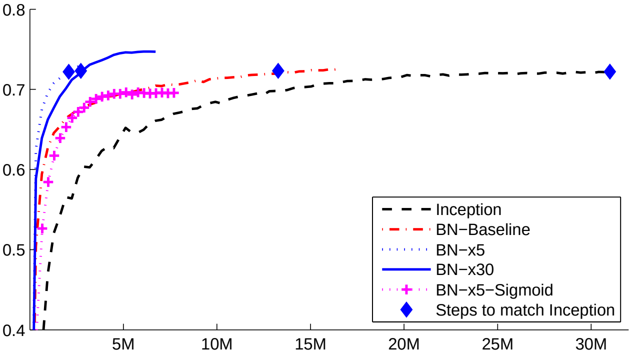
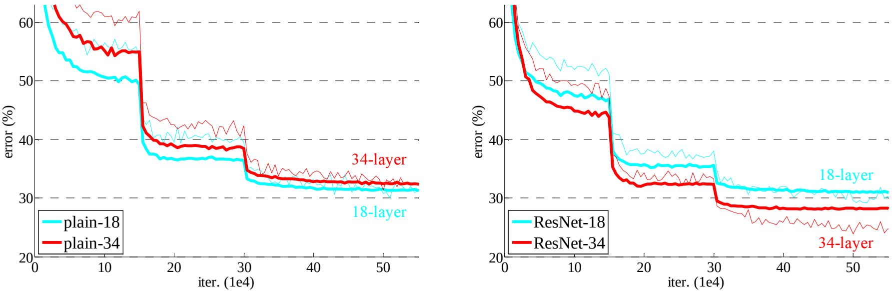


- allows to increase learning rate, remove local response normalization and dropout, and reduce weight decay

residual networks

[He et al. 2016]

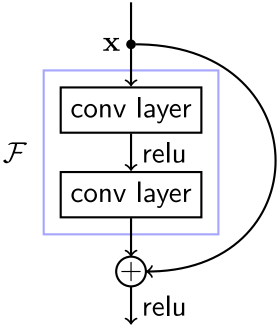


- when initialization, normalization and optimization are appropriately addressed, a degradation is exposed with increasing depth

residual networks

[He et al. 2016]

- “it is easier to push a residual to zero than to fit an identity mapping by a stack of nonlinear layers”



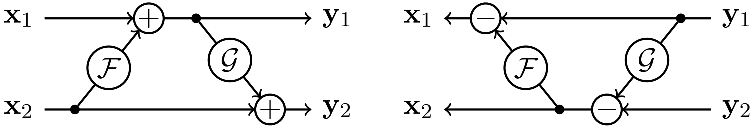
- trained up to 152 layers
- won first place on several ILSVRC and COCO 2015 tasks

He, Zhang, Ren, Sun. CVPR 2016. Deep Residual Learning for Image Recognition.

reversible networks

[Gomez et al. 2017]

- consist of a chain of reversible blocks



$$\mathbf{y}_1 = \mathbf{x}_1 + \mathcal{F}(\mathbf{x}_2)$$
$$\mathbf{y}_2 = \mathbf{x}_2 + \mathcal{G}(\mathbf{y}_1)$$

$$\mathbf{x}_1 = \mathbf{y}_1 - \mathcal{F}(\mathbf{x}_2)$$
$$\mathbf{x}_2 = \mathbf{y}_2 + \mathcal{G}(\mathbf{y}_1)$$

- activations can be recomputed during backward pass
- memory is constant in the number of layers!
- trained up to 600 layers on single GPU

Gomez, Ren, Urtasun, Grosse. 2017. The Reversible Residual Network: Backpropagation Without Storing Activations.

reversible networks

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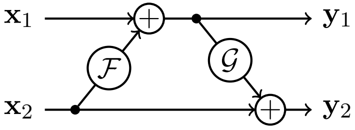

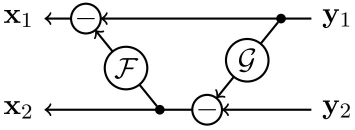

$$\begin{aligned}\mathbf{y}_1 &= \mathbf{x}_1 + \mathcal{F}(\mathbf{x}_2)\\ \mathbf{y}_2 &= \mathbf{x}_2 + \mathcal{G}(\mathbf{y}_1)\end{aligned}$$

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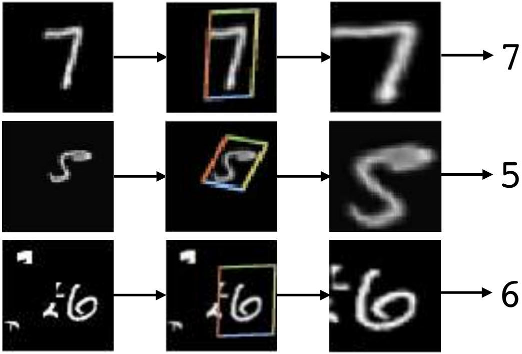
Gomez, Ren, Urtasun, Grosse. 2017. The Reversible Residual Network: Backpropagation Without Storing Activations.

spatial transformer networks

[Jaderberg et al. 2015]

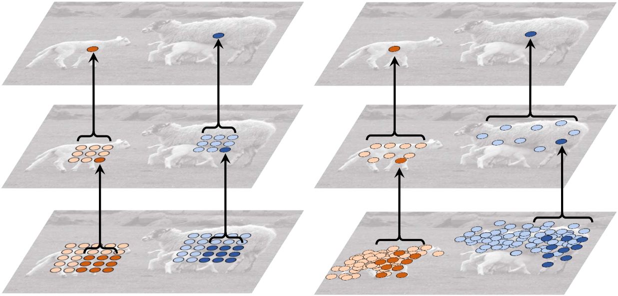


- predict a spatial transformation to localize an object, apply the transformation, resample and classify
- trained end-to-end

Jaderberg, Simonyan, Zisserman, Kavukcuoglu. NIPS 2015. Spatial Transformer Networks.

deformable convolution

[Dai et al. 2017]



- learn to predict offsets used in convolution as a function of the input image
- automatically adjust receptive field per unit

Dai, Qi, Xiong, Li, Zhang, Hu, Wei. 2017. Deformable Convolutional Networks.

deformable convolution

[Dai et al. 2017]

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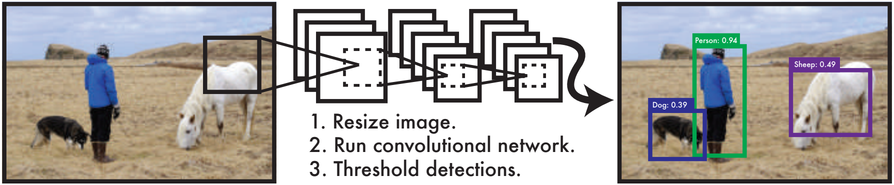
“you only look once”

[Redmon et al. 2016]



- learn to detect objects as a single classification and regression task, without scanning the image or detecting candidate regions
- first object detector to operate at 45fps

Redmon, Divvala, Girshick, Farhadi. CVPR 2016. You Only Look Once: Unified, Real-Time Object Detection.

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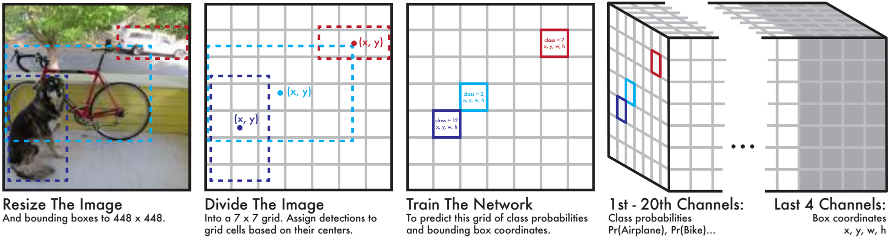

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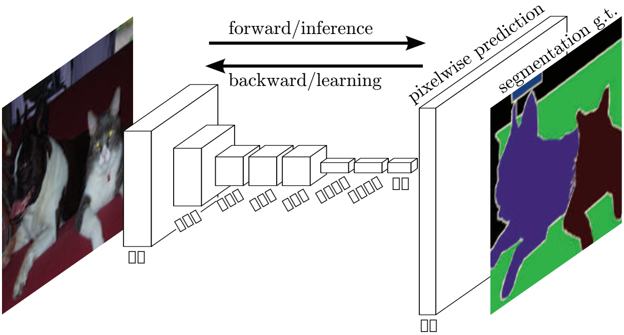
fully convolutional networks

[Long et al. 2015]



- learn to upsample and produce images of the same resolution and the input image
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Long, Shelhamer, Darrell. CVPR 2015. Fully Convolutional Networks for Semantic Segmentation.

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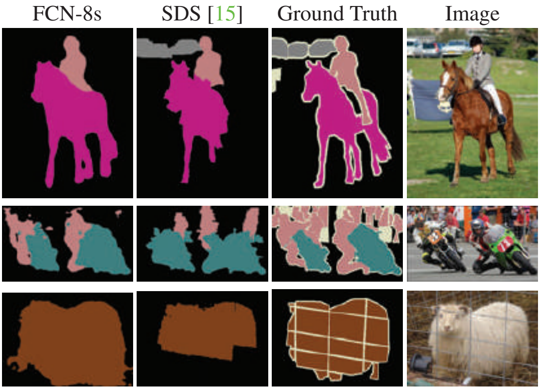

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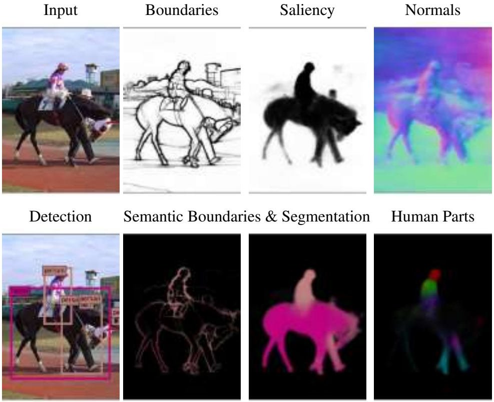
UberNet

[Kokkinos 2017]

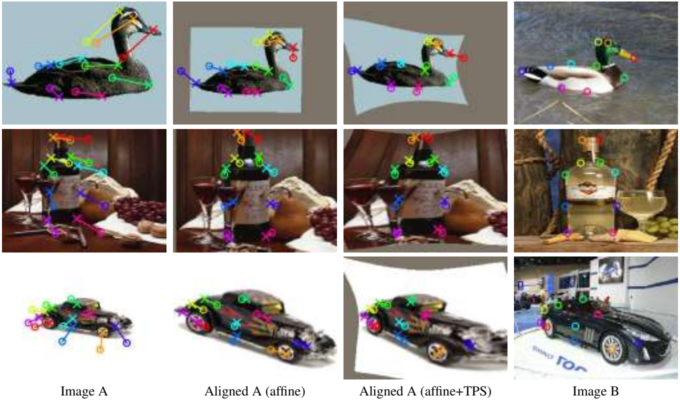


- learn several vision tasks with a joint network architecture including task-specific skip layers

Kokkinos. CVPR 2017. Ubernet: Training a Universal Convolutional Neural Network for Low-, Mid-, and High-Level Vision Using Diverse Datasets and Limited Memory.

geometric matching

[Rocco et al. 2017]



- mimic the standard steps of feature extraction, matching and simultaneous inlier detection and model parameter estimation
- still trainable end-to-end

Rocco, Arandjelovic, Sivic. CVPR 2017. Convolutional Neural Network Architecture for Geometric Matching.

photorealistic style transfer

[Luan et al. 2017]

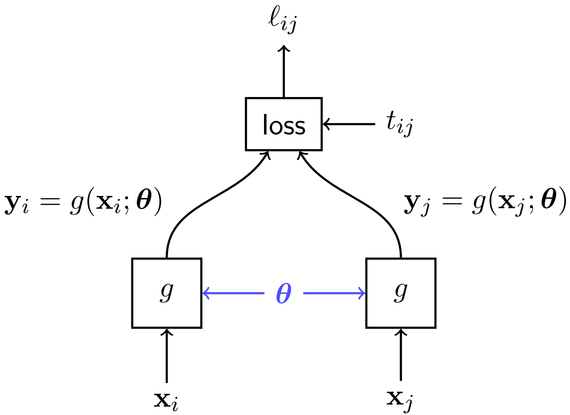
(a) Reference style image (b) Input image (c) Neural Style (distortions) (d) Our result (e) Insets

Luan, Paris, Shechtman, Bala. CVPR 2017. Deep Photo Style Transfer.

unsupervised learning and image retrieval

siamese architecture

[LeCun et al. 2005]



Chopra, Hadsell, Lecun, CVPR 2005. Learning a Similarity Metric Discriminatively, with Application to Face Verification.

manifold learning

[LeCun et al. 2006]

- input samples $\mathbf{x}_i$, output vectors $\mathbf{y}_i = g(\mathbf{x}_i; \boldsymbol{\theta})$
- target variables $t_{ij} = \mathbb{1}[\mathrm{sim}(\mathbf{x}_i, \mathbf{x}_j)]$
- contrastive loss

$$\ell_{ij} = t_{ij} \|\mathbf{y}_i - \mathbf{y}_j\|^2 + (1 - t_{ij})[m - \|\mathbf{y}_i - \mathbf{y}_j\|]_+^2$$

Hadsell, Chopra, Lecun. CVPR 2006. Dimensionality Reduction By Learning an Invariant Mapping.

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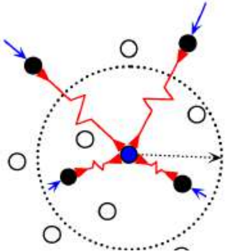
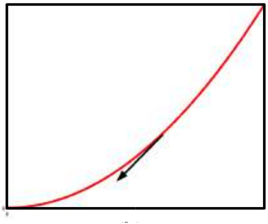
similar





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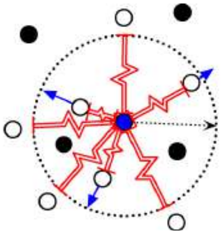
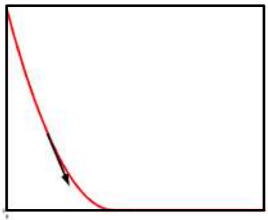
dissimilar





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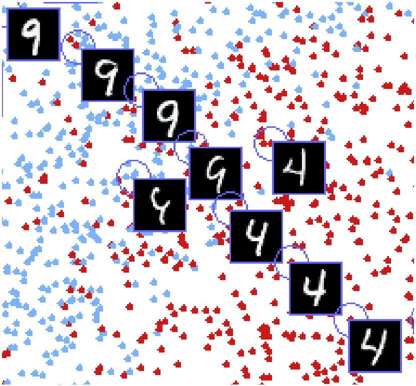
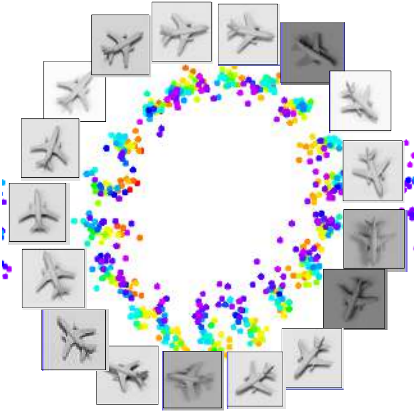
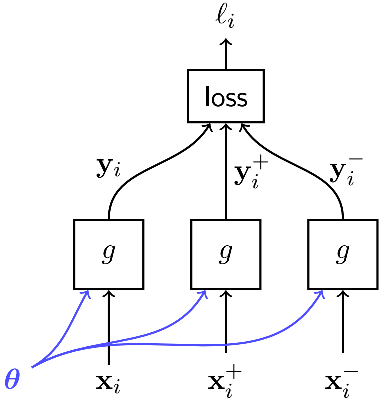




Hadsell, Chopra, Lecun. CVPR 2006. Dimensionality Reduction By Learning an Invariant Mapping.

triplet architecture

[Wang et al. 2014]



Wang, Song, Leung, Rosenberg, Wang, Philbin, Chen, Wu. CVPR 2014. Learning Fine-Grained Image Similarity with Deep Ranking.

learning to rank

[Wang et al. 2014]

- input "anchor" $\mathbf{x}_i$, output vector $\mathbf{y}_i = g(\mathbf{x}_i; \boldsymbol{\theta})$
- positive $\mathbf{y}_i^+ = g(\mathbf{x}_i^+; \boldsymbol{\theta})$, negative $\mathbf{y}_i^- = g(\mathbf{x}_i^-; \boldsymbol{\theta})$
- triplet loss

$$\ell_i = \left[m + \|\mathbf{y}_i - \mathbf{y}_i^+\|^2 - \|\mathbf{y}_i - \mathbf{y}_i^-\|^2\right]_+$$

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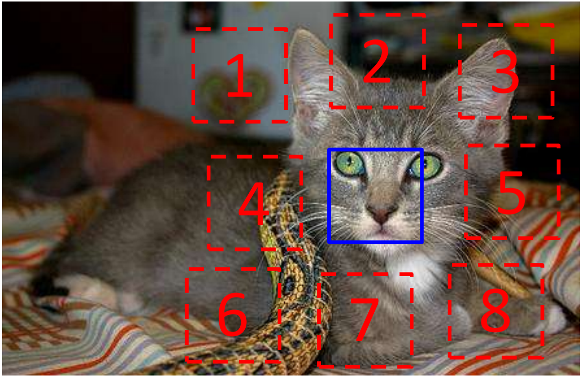
unsupervised learning by solving puzzles

[Doersch et al. 2015]



X = ( , ); Y = 3

Doersch, Gupta, Efros. ICCV 2015. Unsupervised Visual Representation Learning By Context Prediction.

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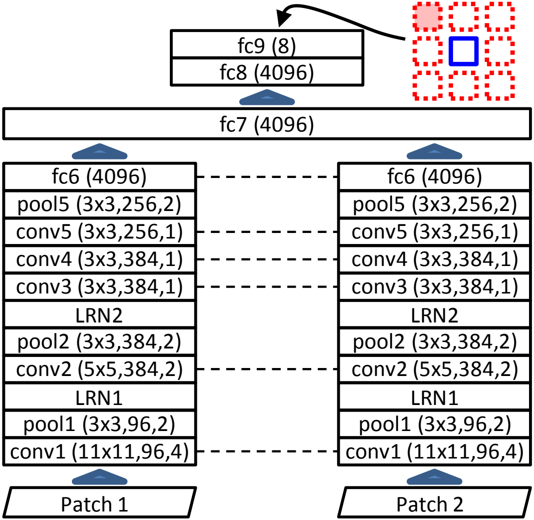

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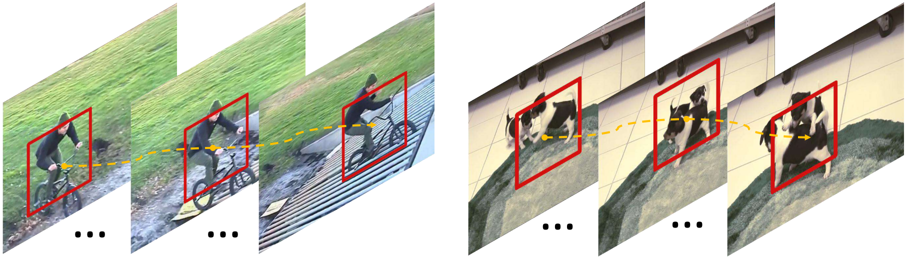

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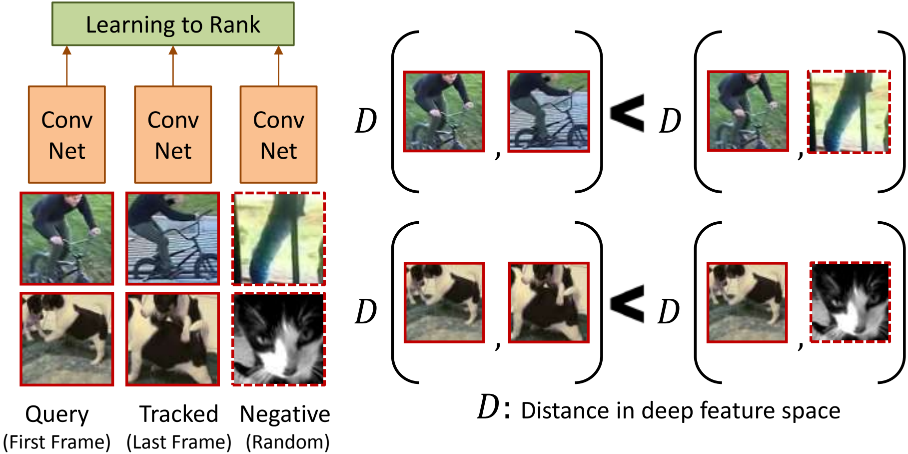

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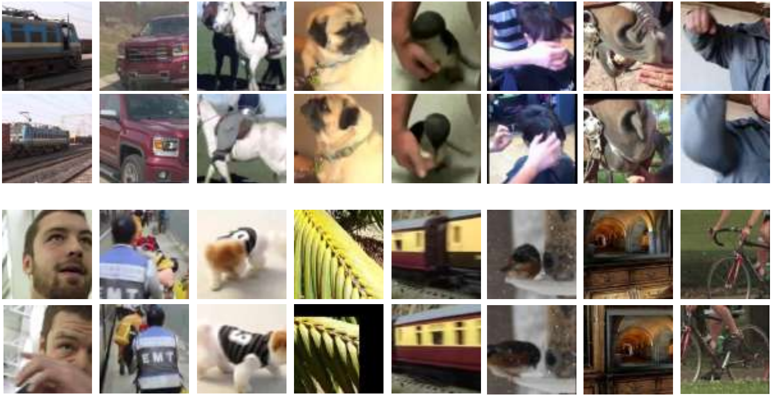
Query
(First Frame)

Tracked
(Last Frame)

Query
(First Frame)

Tracked
(Last Frame)



Wang and Gupta. ICCV 2015. Unsupervised Learning of Visual Representations Using Videos.

ranking by CNN features

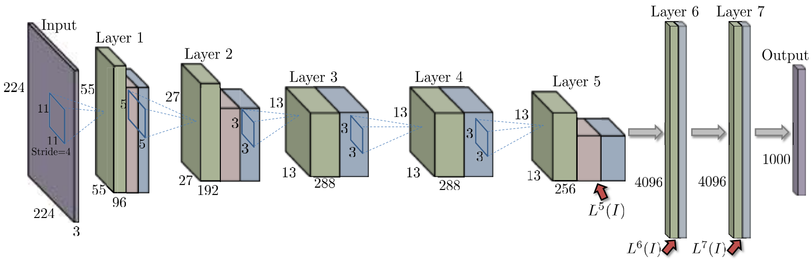
[Krizhevsky et al. 2012]

- use the last fully-connected layer features

Krizhevsky, Sutskever, Hinton. NIPS 2012. Imagenet Classification with Deep Convolutional Neural Networks.

neural codes

[Babenko et al. 2014]



- investigate more than the last fully-connected layer
- fine-tune by softmax on 672 classes of 200k landmark photos

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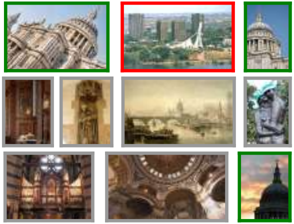
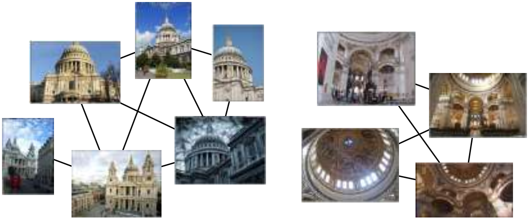
fine-tuning

[Gordo et al. 2016]





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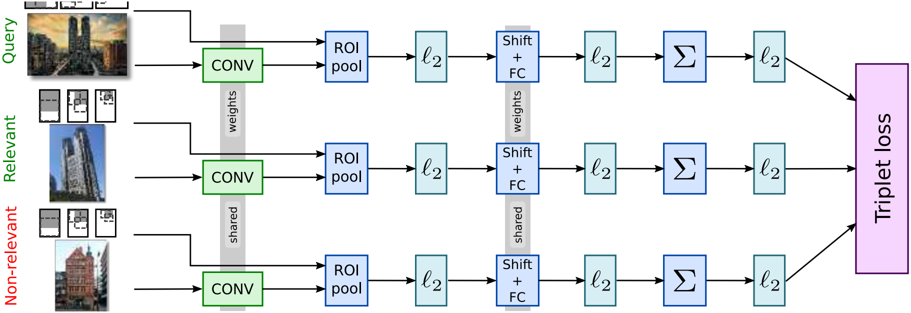

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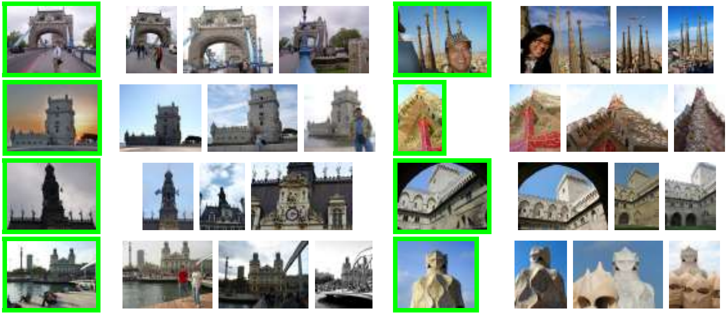
unsupervised fine-tuning

[Radenovic et al. 2016]



(positive)

- reconstruct 700 3d models with 160k images by SfM on 7M images
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Radenovic, Tolias, Chum. ECCV 2016. CNN Image Retrieval Learns From BoW: Unsupervised Fine-Tuning with Hard Examples.

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[Radenovic et al. 2016]

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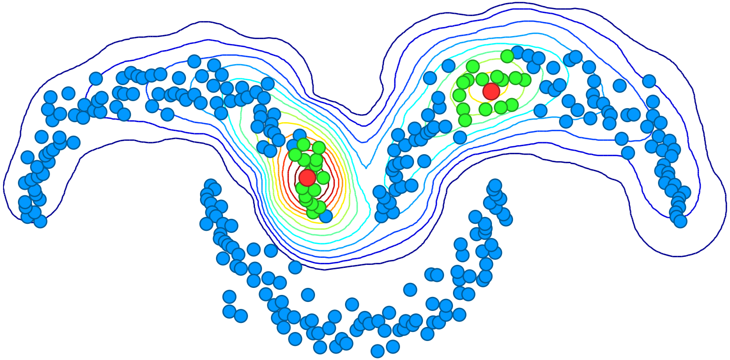
graph-based methods

query expansion and searching on manifolds

[Iscen et al. 2017]



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Iscen, Tolias, Avrithis, Furon, Chum. CVPR 2017. Efficient Diffusion on Region Manifolds: Recovering Small Objects With Compact CNN Representations.

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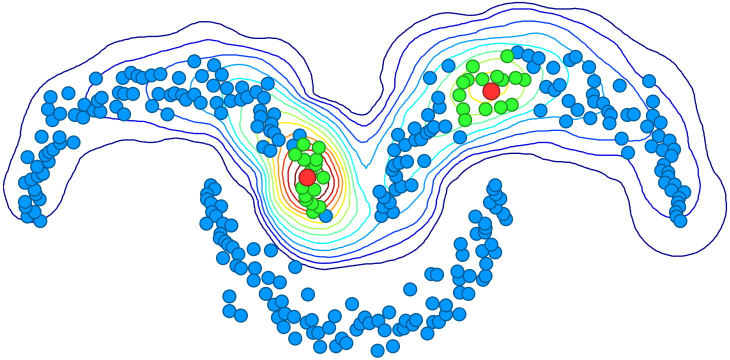

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query expansion as a linear system

[Iscen et al. 2017]

- reciprocal nearest neighbor graph on images or regions
- symmetrically normalized adjacency matrix $\mathcal{W}$
- regularized Laplacian

$$\mathcal{L}_\alpha = \frac{I - \alpha \mathcal{W}}{1 - \alpha}$$

- initial query: sparse observation vector
  $y_i = \mathbb{1}[i \text{ is query (or neighbor)}]$
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searching on manifolds as smoothing

[Iscen et al. 2017]

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$$\mathcal{L}_\alpha^{-1} = h_\alpha(\mathcal{W}) = (1-\alpha)(I - \alpha\mathcal{W})^{-1}$$

- given any matrix function h, we want to compute

$$\mathbf{x} = h(\mathcal{W})\mathbf{y}$$

without computing $h(\mathcal{W})$

Iscen, Tolias, Avrithis, Furon, Chum. arXiv 2017. Fast Spectral Ranking for Similarity Search.

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- low-rank approximation
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searching on manifolds as smoothing

$$\mathbf{x} \approx U \, h\left(\underbrace{\boxed{\Lambda}}_{\text{diagonal}} \right) U^\top \underbrace{\mathbf{y}}_{\text{sparse}}$$

- eigenvalue decomposition of $\mathcal{W}$
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searching on manifolds as smoothing

$$\mathbf{x} \approx \mathcal{F}^{-1} \, h \left(\Lambda \right) \mathcal{F} \, \mathbf{y}$$

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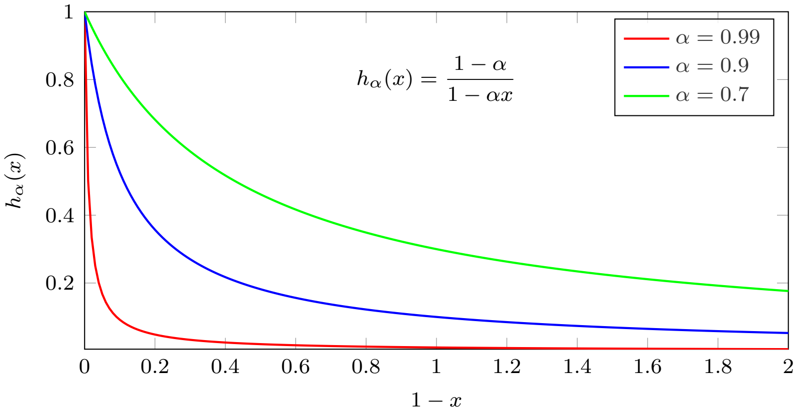

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unsupervised object discovery

[Siméoni et al. 2016]

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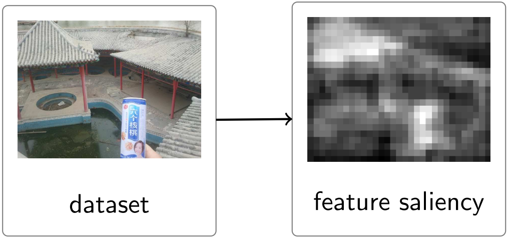

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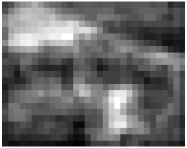
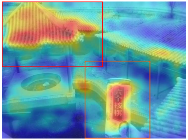


feature saliency



FS regions

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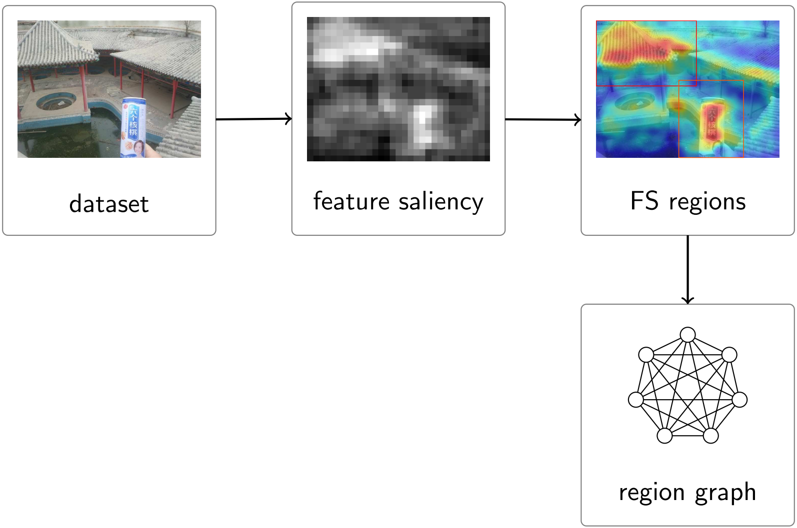

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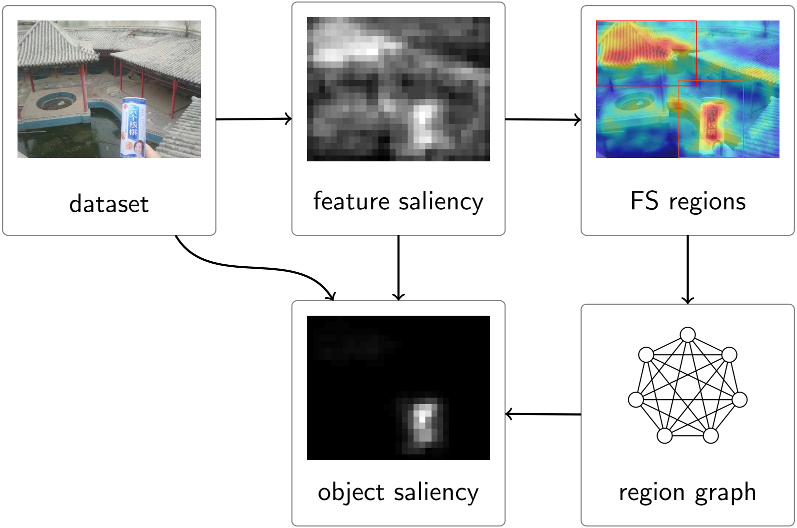

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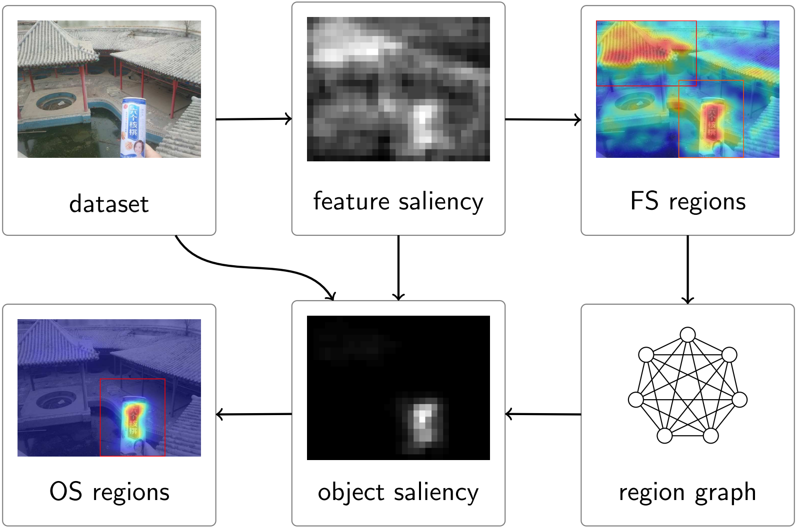
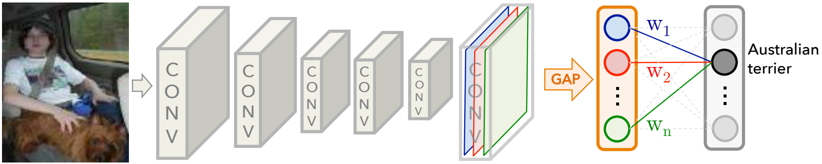


Siméoni, Iscen, Tolias, Avrithis, Chum. arXiv 2017. Unsupervised deep object discovery for instance recognition.

class activation mapping (CAM)

[Zhou et al. 2016]



- global average pooling

$$S_c = \sum_k w_k^c \sum_{x,y} A_k(x, y)$$

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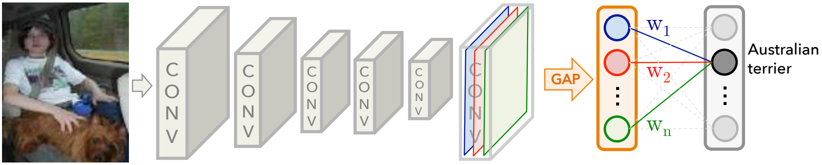

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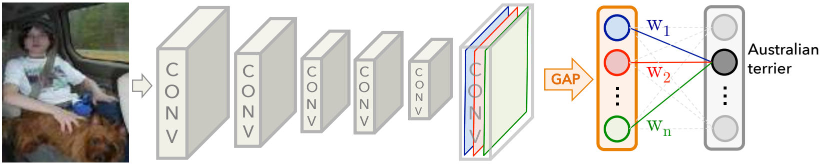

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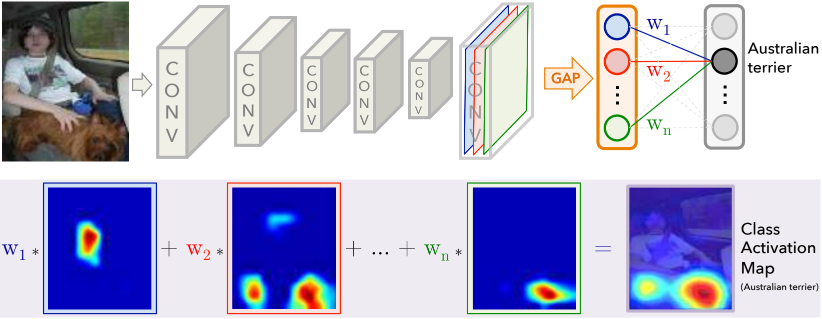

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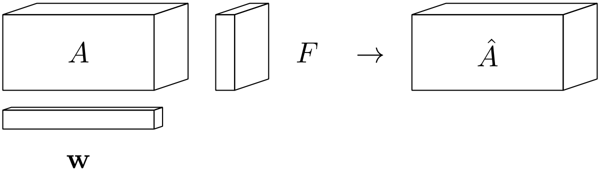
cross-dimensional weighting (CroW)

[Kalantidis et al. 2016]



- spatial weights (visual saliency)

$$F(x, y) = \sum_k A_k(x, y)$$

- channel weights (sparsity sensitive)

$$w_k = -\log\left(\epsilon + \sum_{x,y} \mathbb{1}[A_k(x, y)]\right)$$

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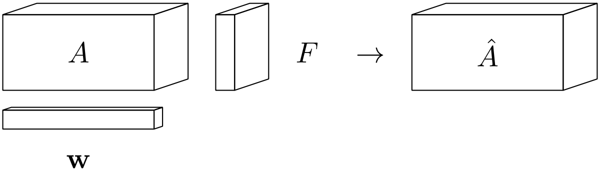

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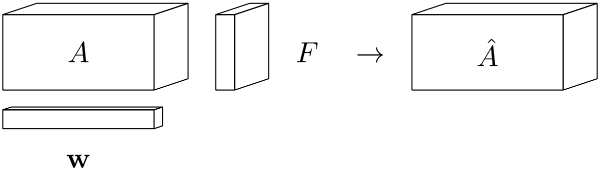

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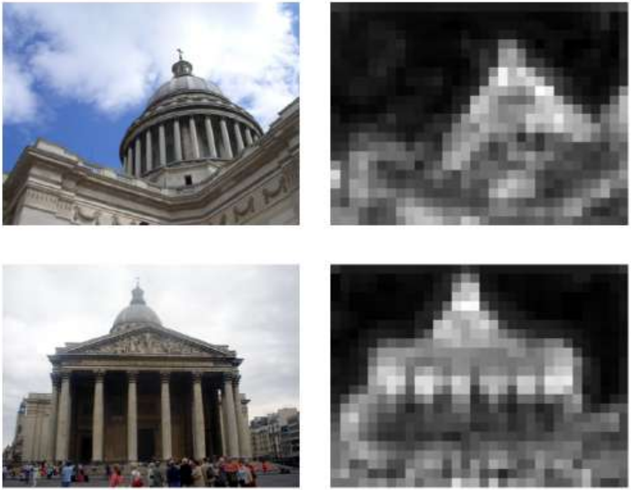

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feature saliency (FS) map

- channel weights (sparsity sensitive, as in CroW)

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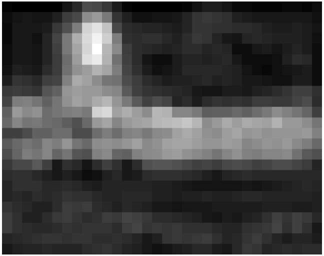
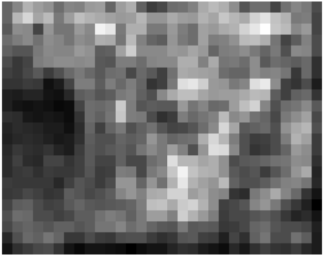
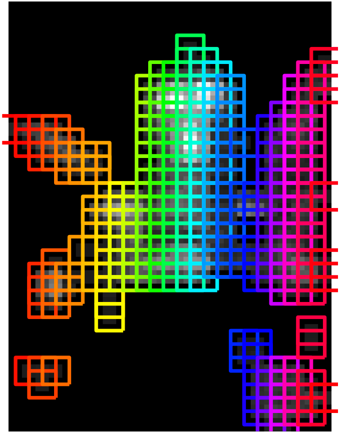




region detection with EGM

[Avrithis and Kalantidis 2012]



- expanding Gaussian mixtures (EGM)
- generalized from points to 2d functions (images)

Avrithis and Kalantidis. ECCV 2012. Approximate Gaussian Mixtures for Large Scale Vocabularies.

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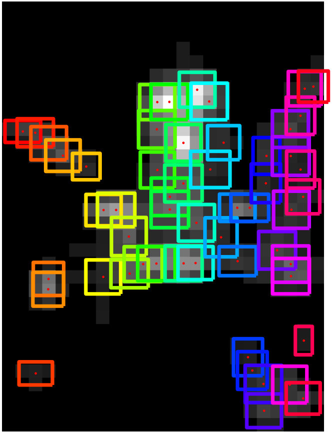

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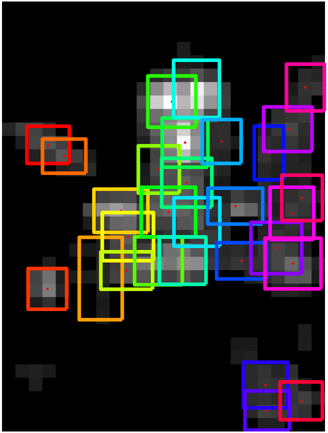

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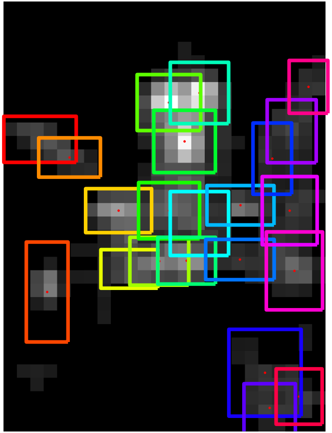

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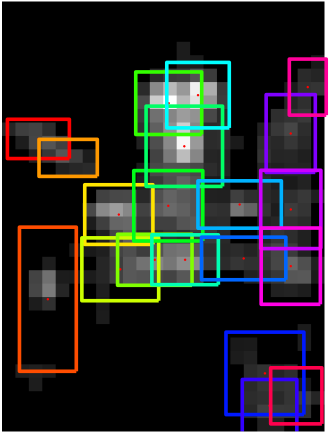

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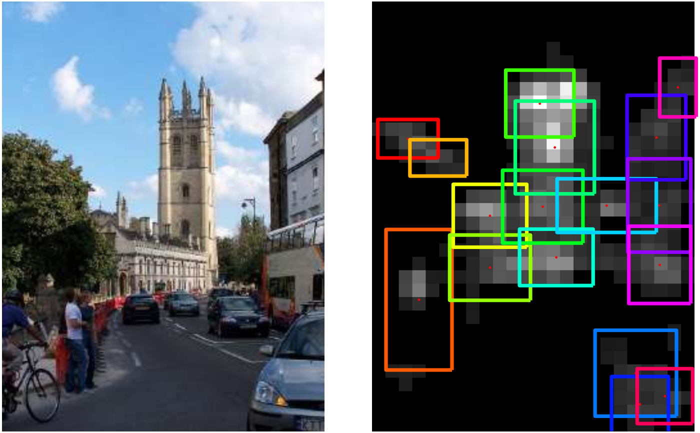

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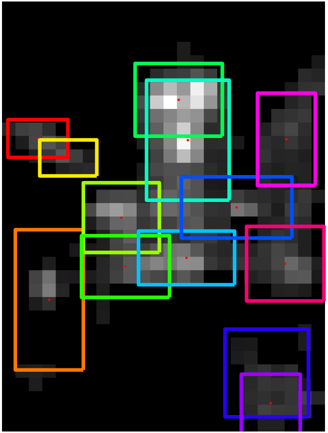

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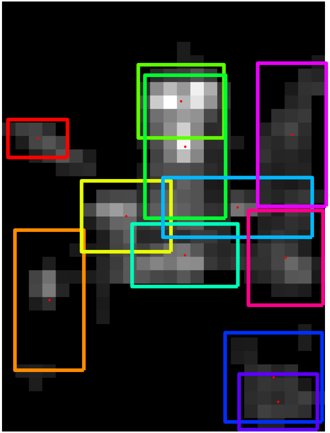

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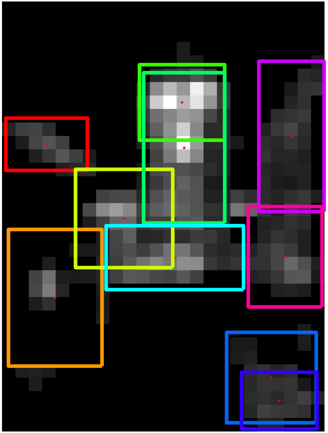

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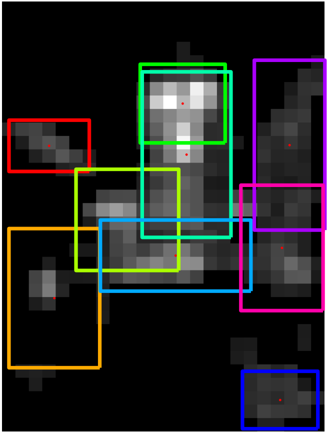

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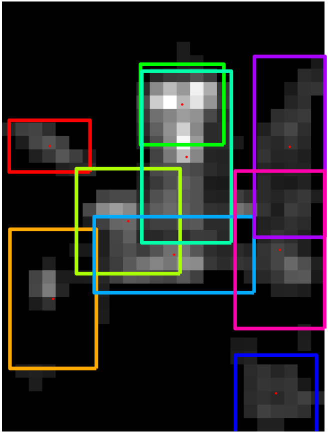

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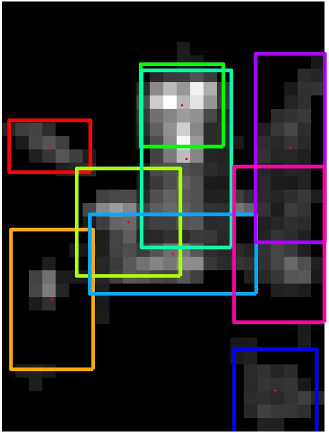

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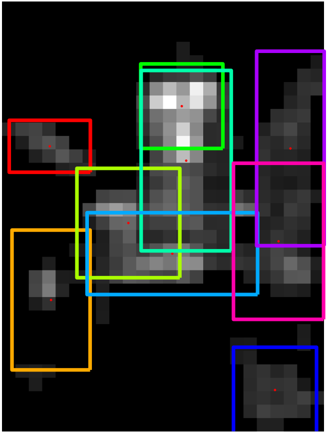

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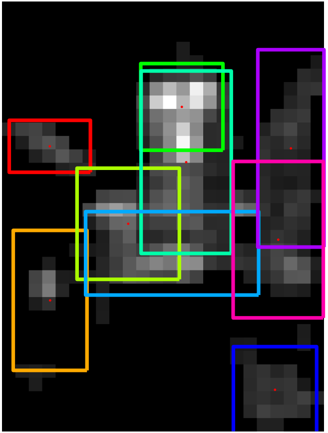

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 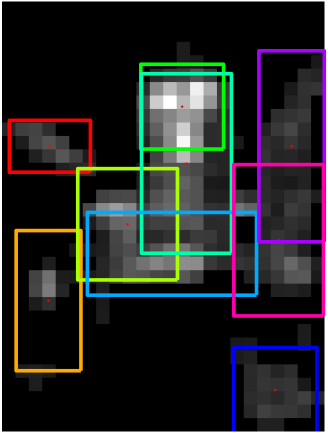

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graph centrality

- construct graph from detected regions
- local search

$$\mathcal{L}_\alpha \mathbf{x} = \mathbf{y}$$

  where $y_i = \mathbb{1}[i \text{ is query}]$
- global centrality (Katz)

$$\mathcal{L}_\alpha \mathbf{g} = \mathbf{1}$$

Katz. Psychometrika 1953. A New Status Index Derived From Sociometric Analysis.

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object saliency (OS) map

$$S(p) = \hat{F}(p) \sum_{i \in N_p} \mathrm{sim}(\mathbf{v}_i, \mathbf{u}_p) f_i g_i$$

object saliency (OS) map

$$S(\underset{\text{patch}}{p}) = \hat{F}(p) \sum_{i \in N_p} \mathrm{sim}(\mathbf{v}_i, \mathbf{u}_p) f_i g_i$$

object saliency (OS) map

$$S(\underset{\text{patch}}{p}) = \hat{F}(p) \sum_{i \in N_p} \mathrm{sim}(\mathbf{v}_i, \underset{\text{descriptor}}{\mathbf{u}_p}) f_i g_i$$

patch

descriptor

object saliency (OS) map

$$S(\underbrace{p}_{\text{patch}}) = \hat{F}(p) \underbrace{\sum_{i \in N_p}}_{\text{neighbors}} \mathrm{sim}(\mathbf{v}_i, \underbrace{\mathbf{u}_p}_{\text{descriptor}}) f_i g_i$$

object saliency (OS) map

similarity

$$S(p) = \hat{F}(p) \sum_{i \in N_p} \mathrm{sim}(\mathbf{v}_i, \mathbf{u}_p) f_i g_i$$

patch

neighbors

descriptor

object saliency (OS) map

similarity FS

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patch neighbors descriptor

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centrality

patch descriptor

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patch neighbors descriptor

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object saliency (OS) map

image

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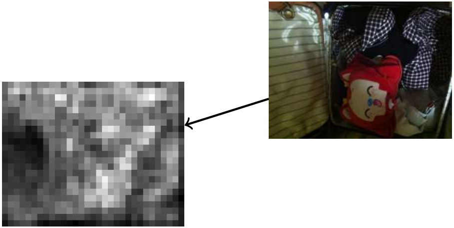

$\hat{F}$

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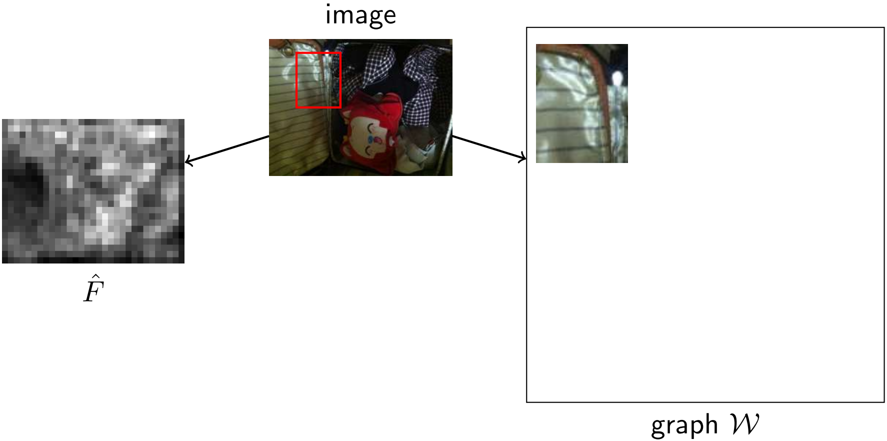

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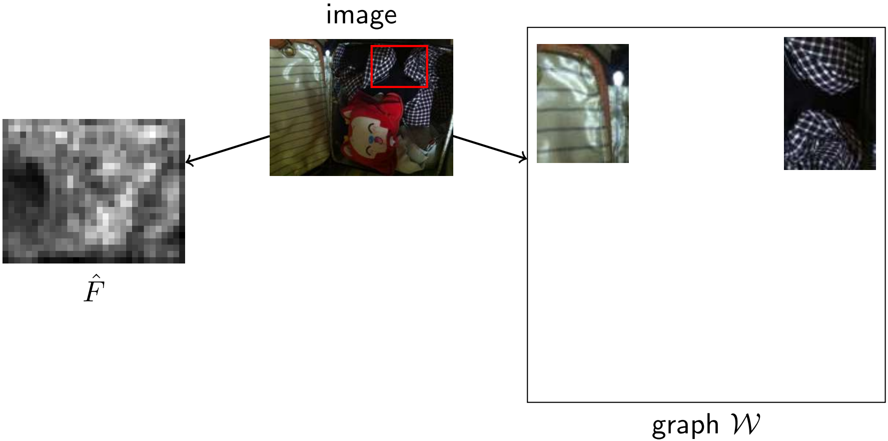

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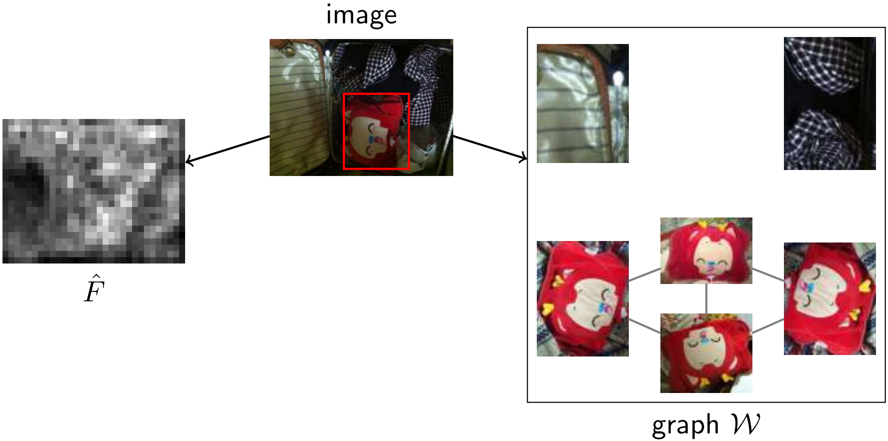

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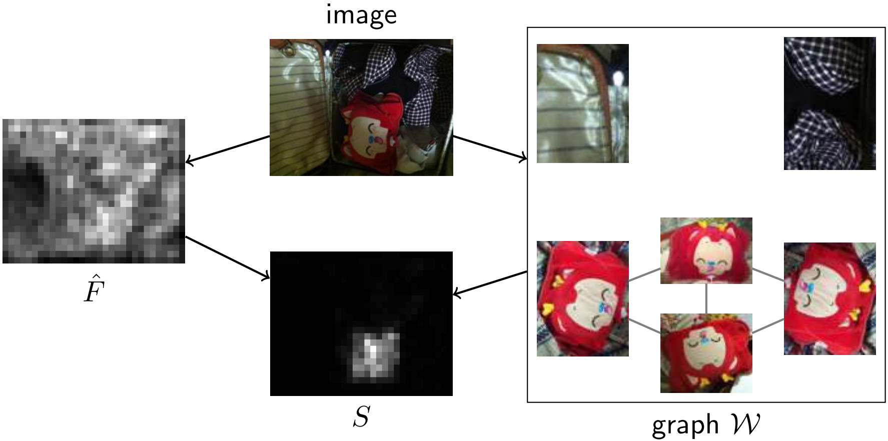
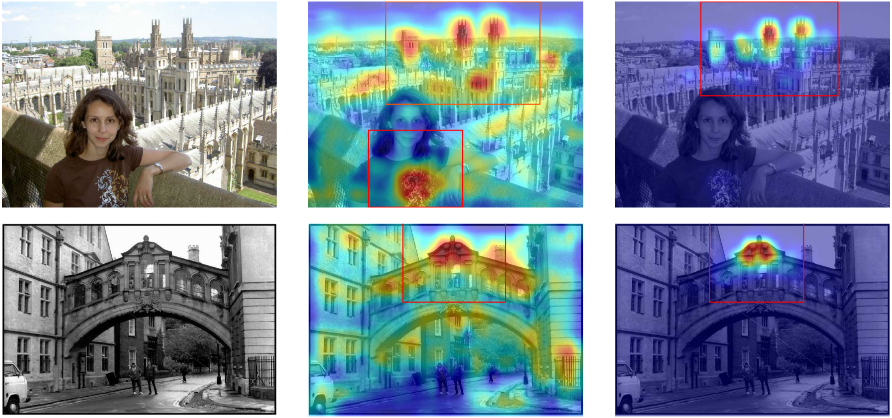
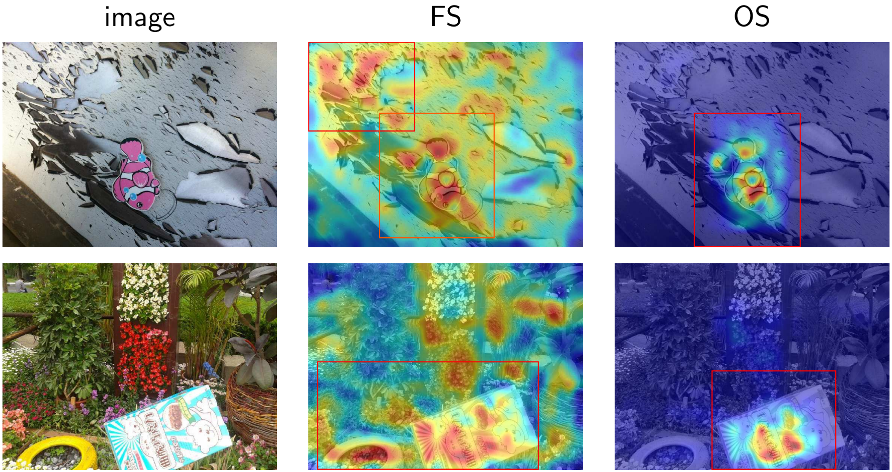


FS versus OS (Oxford 5k)

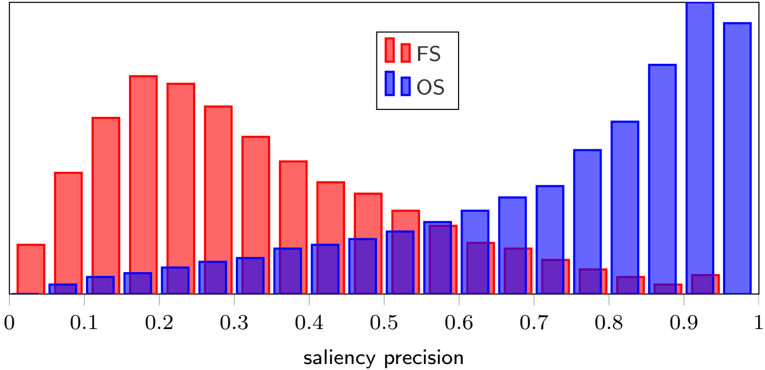
image FS OS



FS versus OS (INSTRE)



what does OS find?

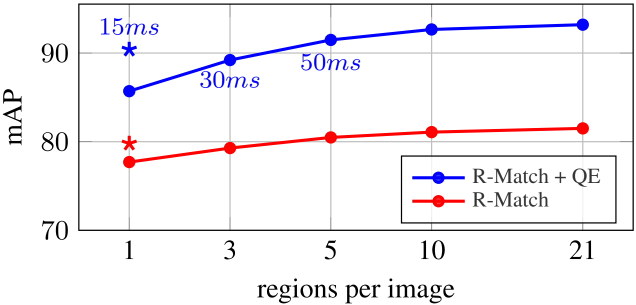


- precision: sum of saliency over ground truth regions, normalized by the sum over the entire image

global image representation

- fine-tuned VGG features [Radenovic *et al.* 2016]
- compute FS, detect regions with EGM and construct graph
- compute OS for each image in the dataset
- re-detect regions with EGM
- max-pool over regions, sum-pool globally as in R-MAC

global versus regional



- regional search: $O(n)$ space and $O(n^2)$ query time, where n is the number of regions (descriptors) per image
- same performance with 5 times less memory and ≈ 4 times faster

state of the art (global)

Method	QE	Instre	Oxford	Oxford105k
MAC	-	48.5	79.7	73.9
R-MAC	-	47.7	77.7	70.1
FS.EGM *	-	48.4	77.5	70.2
OS.EGM *	-	50.1	79.6	71.8
OS.EGM-$\triangle^\star$	-	53.7	79.8	71.4
MAC	✓	71.8	87.4	86.0
R-MAC	✓	70.3	85.7	82.7
FS.EGM *	✓	71.2	89.8	87.9
OS.EGM *	✓	72.7	**90.4**	**88.0**
OS.EGM-$\triangle^\star$	✓	**75.4**	90.1	84.3

- always better than R-MAC, up to 6% at large scale
- compete MAC, even though network was optimized for that
- most gain with QE

summary

- let's go and learn with as little supervision as possible!

joint work with


Oriane Siméoni


Ahmet Iscen


Giorgos Tolias


Teddy Furon


Ondrej Chum

unsupervised object discovery
https://arxiv.org/abs/1709.04725

fast spectral ranking
https://arxiv.org/abs/1703.06935

diffusion on region manifolds
https://arxiv.org/abs/1611.05113

thank you!